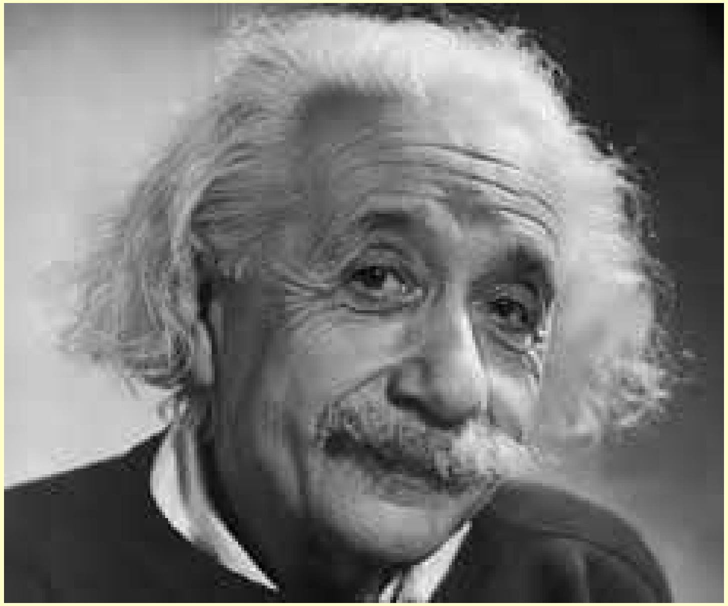
Einstein,
$E=mc^2$,
and Nuclear Processes on Earth
and in the Cosmos

Stanley Yen, TRIUMF

There are probably two physicists who have attained pop-star status in the public mind.



Einstein was a genius who accomplished many things

1905 Theory of Brownian motion – evidence for existence of atoms

Today's first lecture

1905 Photoelectric effect – light comes in discrete bundles of energy

→ quantum theory

After Christmas

1905 Special Relativity – speed of light c as a universal speed limit

- new rules for behaviour of objects moving near c
- equivalence of mass and energy $E = mc^2$

November lectures

1915 General Relativity – warping of space & time by gravity

→ Black Holes, cosmology & expanding universe

After Christmas

In the public mind, the formula

$$E=mc^2$$

is inextricably associated with nuclear energy.

USS Enterprise,
the first nuclear-powered
aircraft carrier

Today's lecture:

Where did this formula $E=mc^2$ come from?

What does it mean?

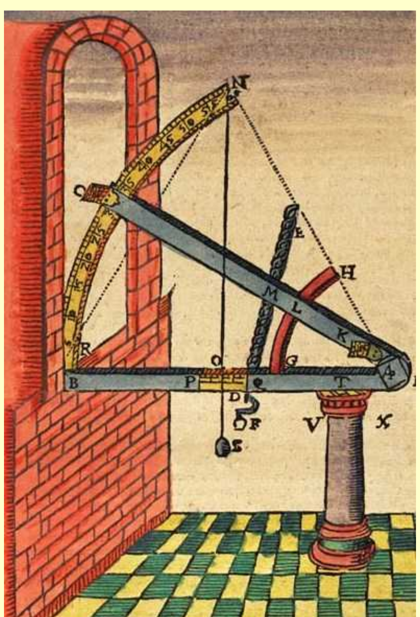
Why is it associated with nuclear energy? (It actually applies to all forms of energy)

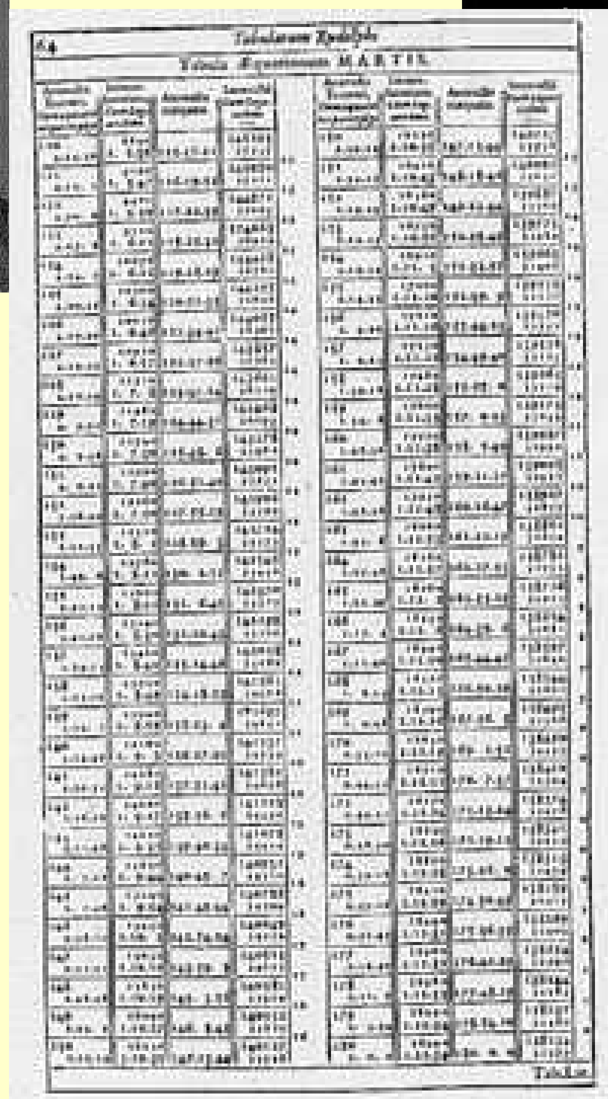
Applications to nuclear power, medicine, astronomy

Two kinds of scientists:

Experimentalists – build apparatus
and make observations

Tycho Brahe

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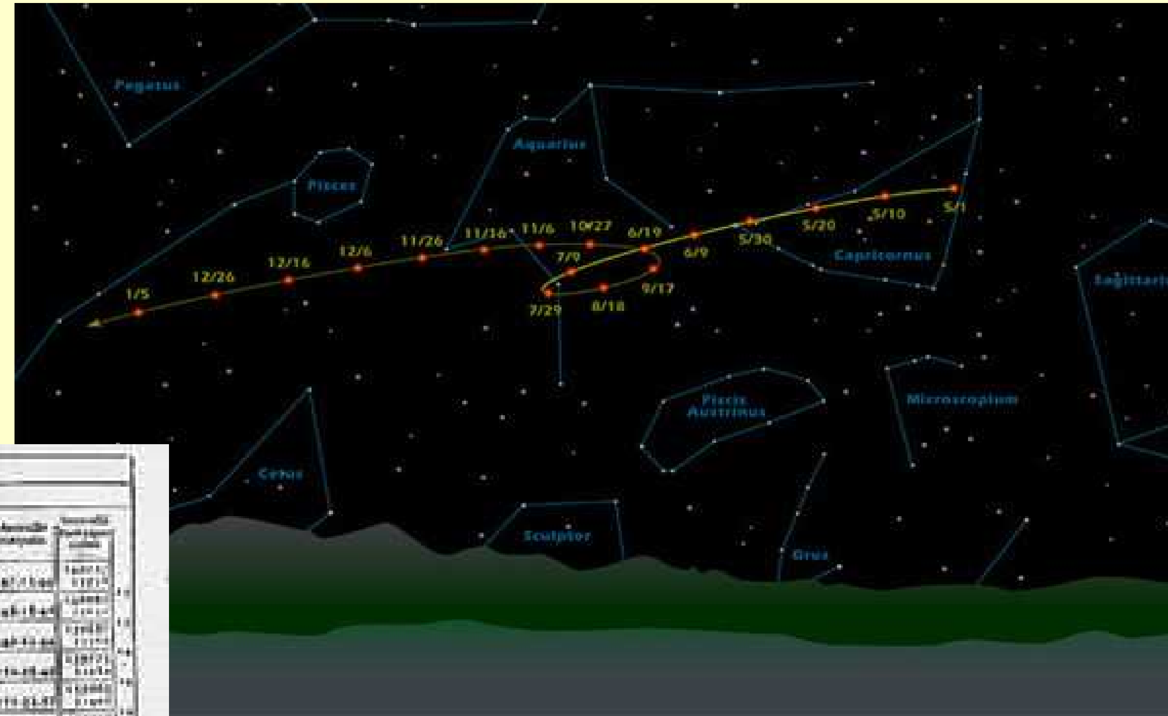
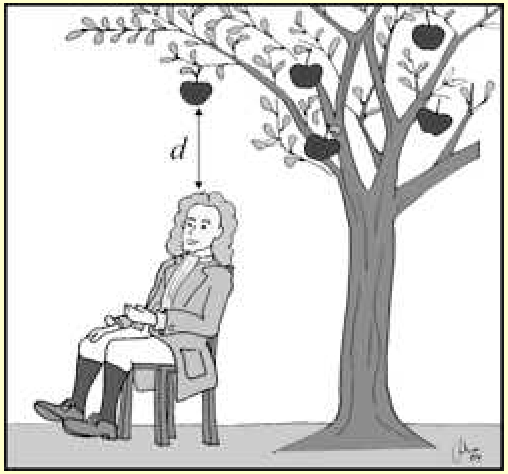
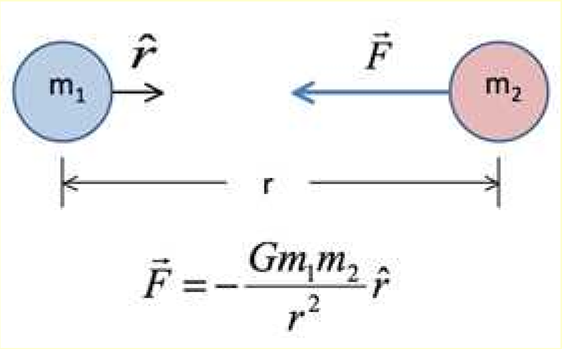
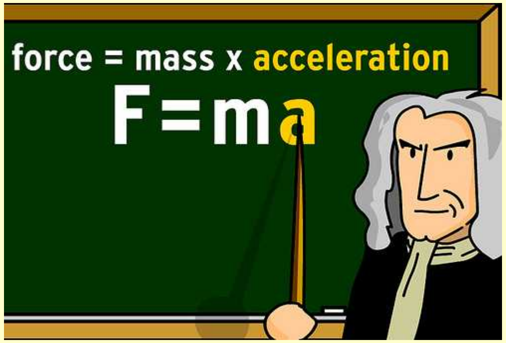


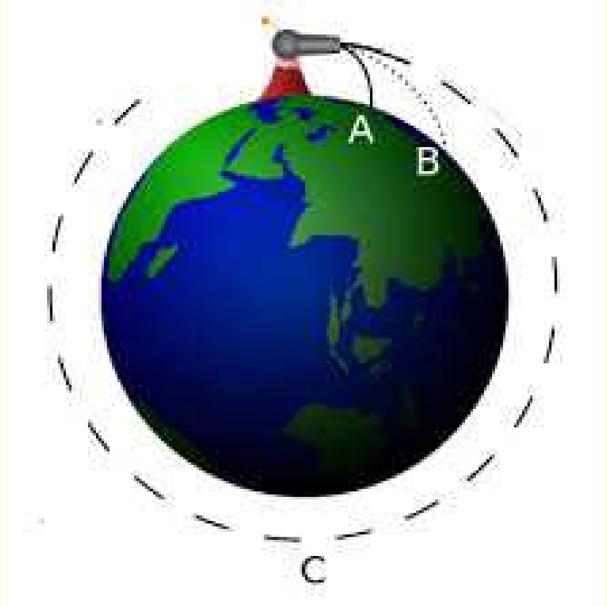
Two kinds of scientists: Theorists – find a concise mathematical framework to explain the observations

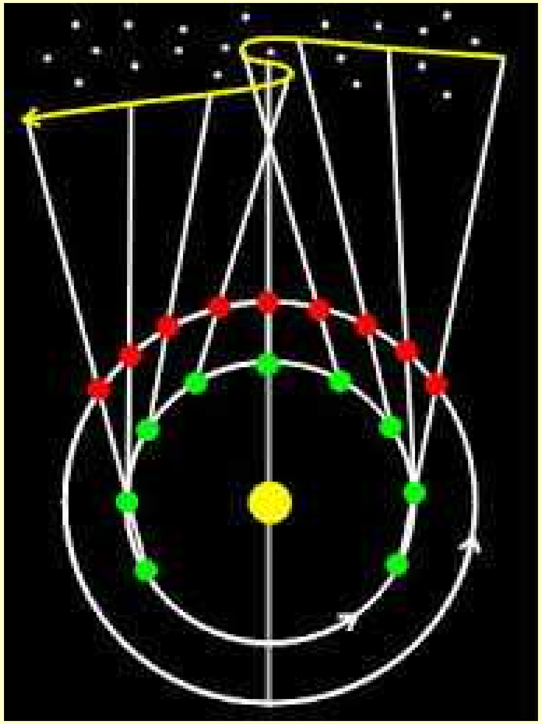
Isaac Newton











Newton's laws of motion → “classical mechanics”, which successfully explained the behaviour of cannonballs, motion of trains and pendulum clocks, planetary motion. Until Einstein, this was believed to be the complete truth.

Some important formula from classical mechanics that you probably remember from high school physics:

$F = ma$	Force = mass x acceleration
$p = mv$	Momentum of a moving object
$E = \frac{1}{2} m v^2$	Kinetic energy of a moving object

- Conservation of momentum
- Conservation of energy

Underlying assumptions:

The mass of an object doesn't depend on how fast it's moving.

The size of an object doesn't depend on how fast it's moving.

The passage of time is the same, no matter how fast a clock is moving.

In Newton's laws of mechanics, the mass m of an object was unrelated to how much energy E it had.

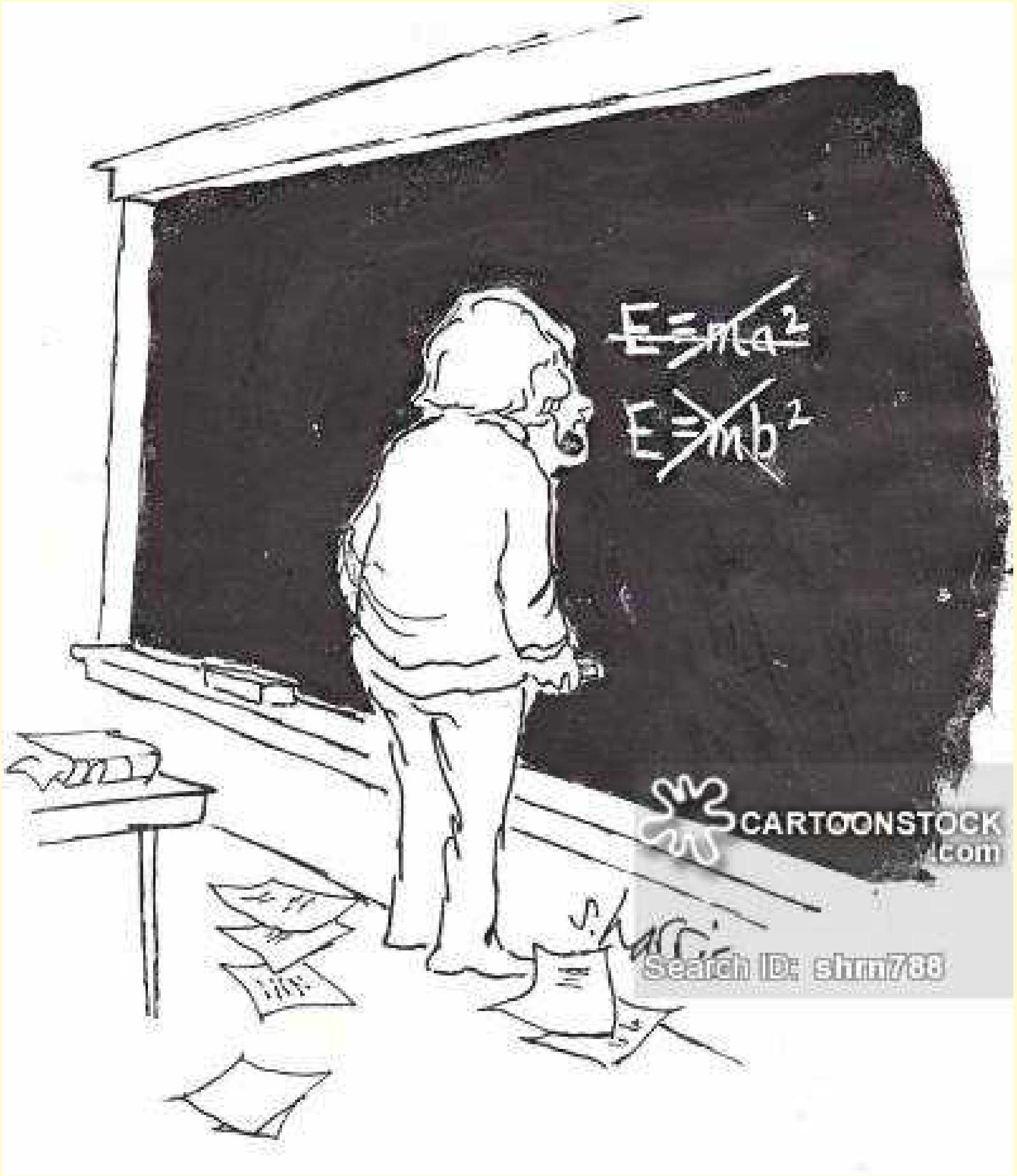
cold horse-shoe

red-hot horse-shoe

But Einstein's new formula $E = mc^2$ says that mass is equivalent to energy, and you can change energy into mass, or mass into energy ..... something entirely new and unknown in Newton's mechanics, and something unknown from everyday experience.

Just like Newton, Einstein was also a theorist ...
he used his mind to discover new laws of physics in order to explain the observed data.

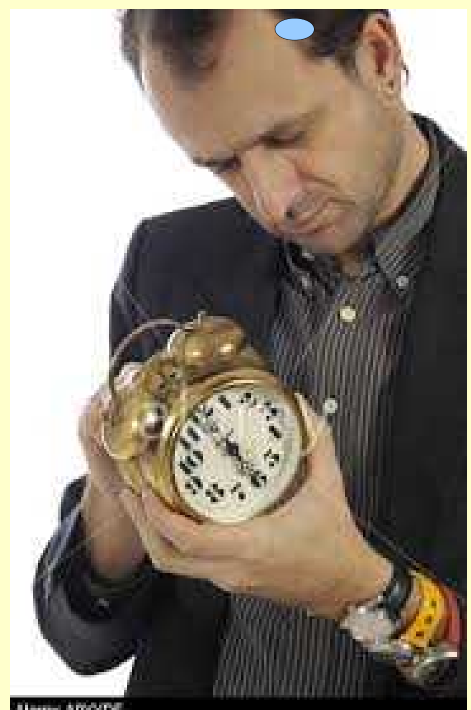
How was this formula $E=mc^2$ discovered?



not like this!

E=mc^2 also not discovered experimentally like this!

Let's add some energy by winding up the spring ...



1.0 kilograms

1.2 kilograms !!

NOT !

Rather, $E=mc^2$ was discovered as part of Einstein's theory of special relativity, when Einstein was seeking to explain the behaviour of light as seen by observers in different reference frames moving with respect to each other.

See next month's lectures - Special relativity explained in detail!

Startling new predictions:

- the speed of light in vacuum is a universal speed limit; nothing can go faster than light

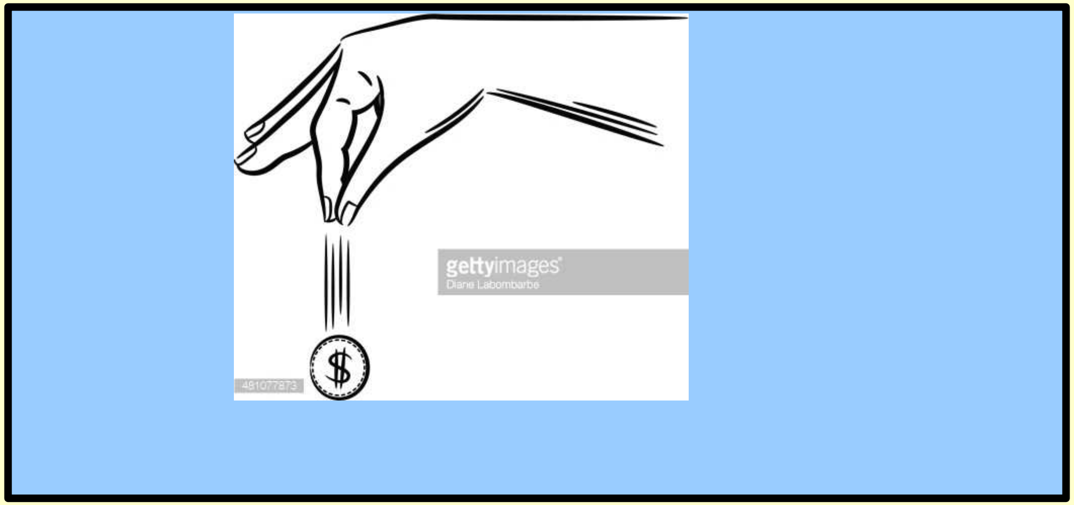
- the apparent mass of an object increases as it approaches the speed of light

- the length of an object shrinks as it approaches the speed of light

- a clock runs slower as it approaches the speed of light

The laws of physics must
the same for observers
in any inertial reference
frame (reference frames
in uniform straight-line motion
with respect to each other).

If I am inside a train with the windows blinds lowered, how could I tell whether the train is moving uniformly in a straight line, or standing still?



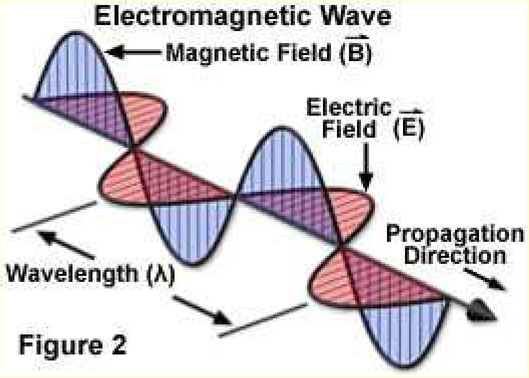
In each case, a dropped coin falls straight down an hits your feet, no matter whether the train is stationary, or moving at 500 km/hr.

and if I measure the force F and the acceleration a, then F=ma, regardless of stationary or moving.

The principle of relativity is that the laws of physics (e.g. F = ma) must be the same, regardless of whether you are at rest or in a uniformly moving frame of reference.

Is it also true that light behaves the same way in any inertial reference frame?

Light (and other electromagnetic waves, like radio waves) are waves, like waves on a pond. The strength of the electric and magnetic fields go up and down, just like the height of the water at the surface of the pond goes up and down.



But in the case of light waves, waving in what? What is the medium, analogous to water for waves on the surface of a pond?

19th century physicists thought that there was some invisible, mysterious medium called “ether” which pervaded all of space, and in which the light waves were “waving”.

If so, then we should be able to detect the motion of the Earth as it moves through this “ether”. All such experiments failed.

Light travels very fast. In a vacuum (e.g. outer space) light travels at

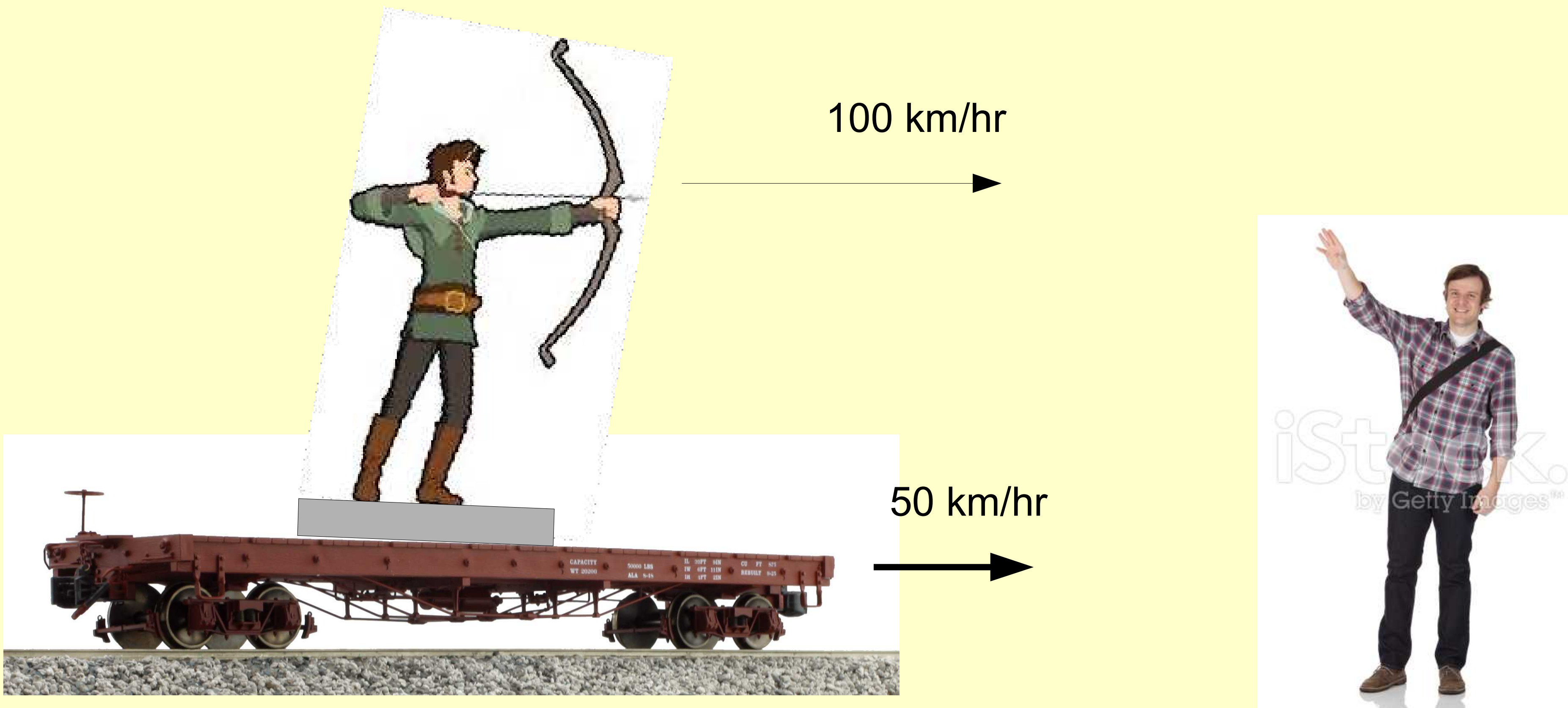
$c = 3 \times 10^8$ metres/second. This value is predicted by the equations of electricity and magnetism that were known at that time (Maxwell's equations).

Once around the Earth	1/7 second
Earth to Moon	2 seconds
Earth to Sun	8 minutes
Earth to nearest star	4.3 years
Earth to nearest galaxy	2.2 million years

Back in the time when physicists thought that “ether” existed, the speed of light was thought to be measured relative to this “ether”.

How do you add velocities in classical mechanics?

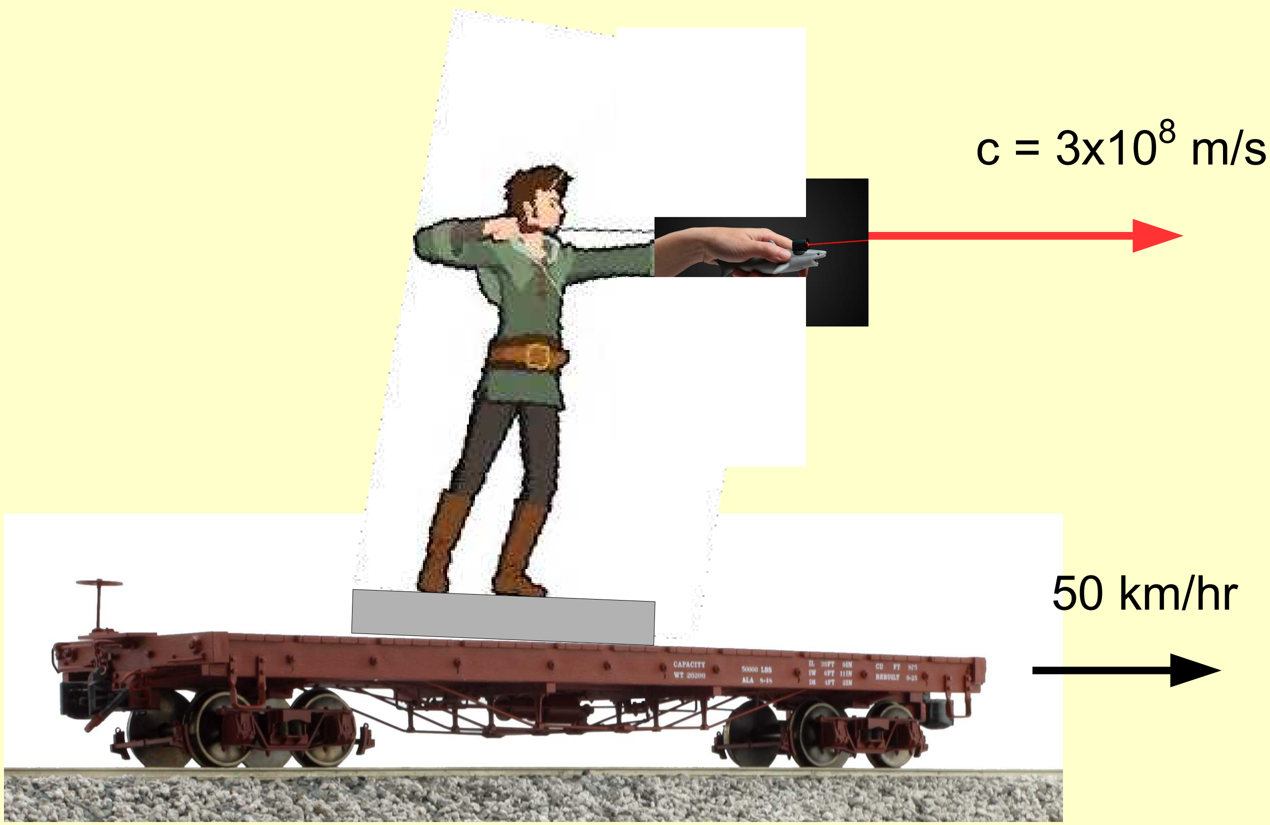
Suppose a train is moving at 50 km/hr, and someone on the train fires an arrow moving at 100 km/hr relative to the train.
How fast is the arrow moving relative to a person on the ground?



Classical mechanics says $v_{total} = v_1 + v_2 = 100 + 50 = 150$ km/hr

But what happens if one of the velocities is the speed of light itself?

Suppose a train is moving at 50 km/hr, and someone on the train fires a laser beam moving at c relative to the train.
How fast is the laser beam moving relative to a person on the ground?

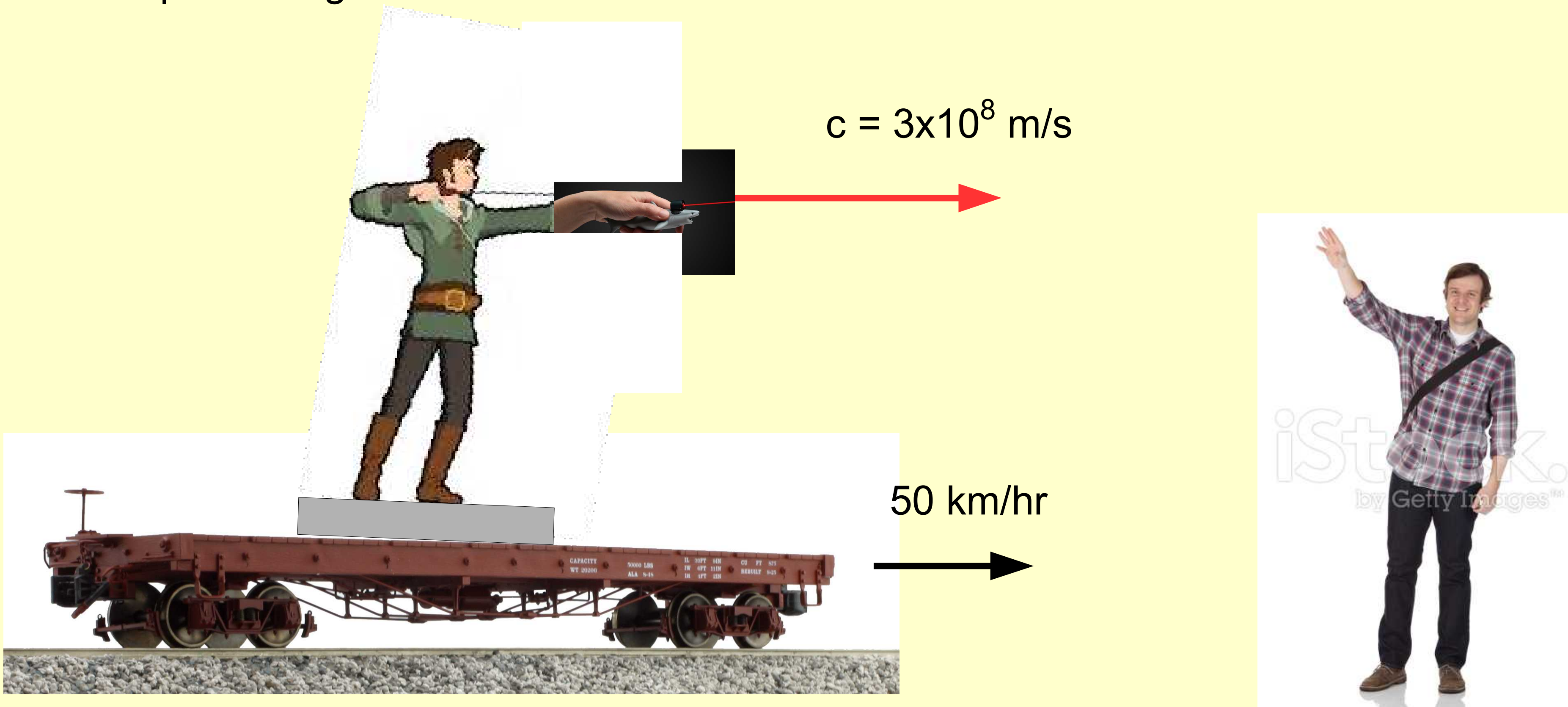


Classical mechanics says the observer on the ground measures
$v_{total} = v_1 + v_2 = c + 50 >$ speed of light in vacuum

Einstein's special relativity says NO ! If there is no “ether”, then there is no absolute reference frame against which to measure the speed of light.

Both the observer riding on the train, and the observer standing still on the ground, will measure exactly the same speed for the light, namely $c = 3 \times 10^8$ metres/second.

The speed of light is a universal constant for all observers.



The stipulation that the speed of light must be a universal constant value for all observers, no matter whether they are moving or at rest,

PLUS the stipulation that the laws of physics must be the same for all observers, stationary or moving, forces a re-write of Newton's laws of motion.

Momentum $p = mv$ becomes $$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

For very slow speeds $v \approx 0$ and $p = mv$ like before.

Energy $E = ½\, mv^2$ becomes $$E = \sqrt{p^2 c^2 + m^2 c^4}$$

For very slow speeds $v \approx 0$, $p \approx 0$
and $E \approx mc^2 + ½\, mv^2$

This is something new: even when an object is at rest ($v = 0$) it has some energy $E = mc^2$

For an object at rest, $E = mc^2$.   We can divide both sides of this equation by c^2 to get

$$m = \frac{E}{c^2}$$

So, even at rest, an object of mass m has an energy content $E = mc^2$.

The greater the energy content of an object, the larger its mass.

This is contrary to everyday experience!

We don't see this in normal life because the effect is so small.

Example: a mechanical system

1 kg alarm clock mass $m = 1$ kg

Suppose I expend 1 Joule of energy to wind it up

so $\Delta E = 1$ Joule

How much does the mass change by?

$\Delta m = \Delta E / c^2 = 1\ \text{J} / (3\text{x}10^8\ \text{m/s})^2 = 1.1 \text{ x } 10^{-17}$ kg

which is so tiny a change that not even the best scale in the world could measure it!

How about a chemical reaction?

Thermite: a highly exothermic reaction used to weld steel rails together

$Fe_2O_3 + 2Al \rightarrow 2Fe + Al_2O_3$

160 g 54 g

214 g

Heat released (enthalpy) ΔE = -851,500 Joules

Since the energy is lost, the mass should decrease.

Change in mass $\Delta m = \Delta E / c^2 = 10^{-9}$ g compared to the 214 g of the initial ingredients

i.e. the mass decreases by about 1 part in 200 billion
impossible to measure with any laboratory balance

That's why, in high school chemistry class, you are taught to balance chemical equations using "conservation of mass", that is, the mass of the ingredients on both sides of a chemical equation must balance each other.

Strictly speaking, it doesn't quite balance, but the difference is so miniscule that you could never measure it.

How about nuclear energies?

Nuclear energies are **much larger** than chemical energies

Chemical reactions involve typical energies of kiloJoules/mole, e.g.

$C + O_2 \rightarrow CO_2 \uparrow$ releases 393 kJoules/mole

Nuclear reactions involve typical energies that are about 1-10 million times larger

$$^{1}H + {}^{1}H + {}^{1}H + {}^{1}H \longrightarrow {}^{4}He + 2\,e^{+} + 2\nu$$

releases 2.5 billion kJoules/mole

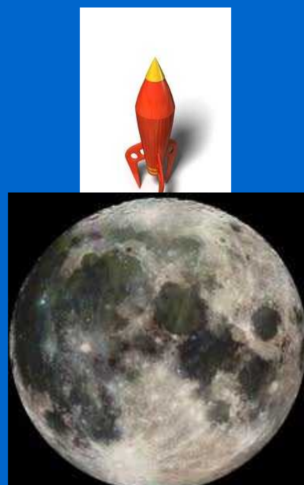
about 6 million times more energy per mole than the chemical reaction

the reaction that produces energy in the Sun!

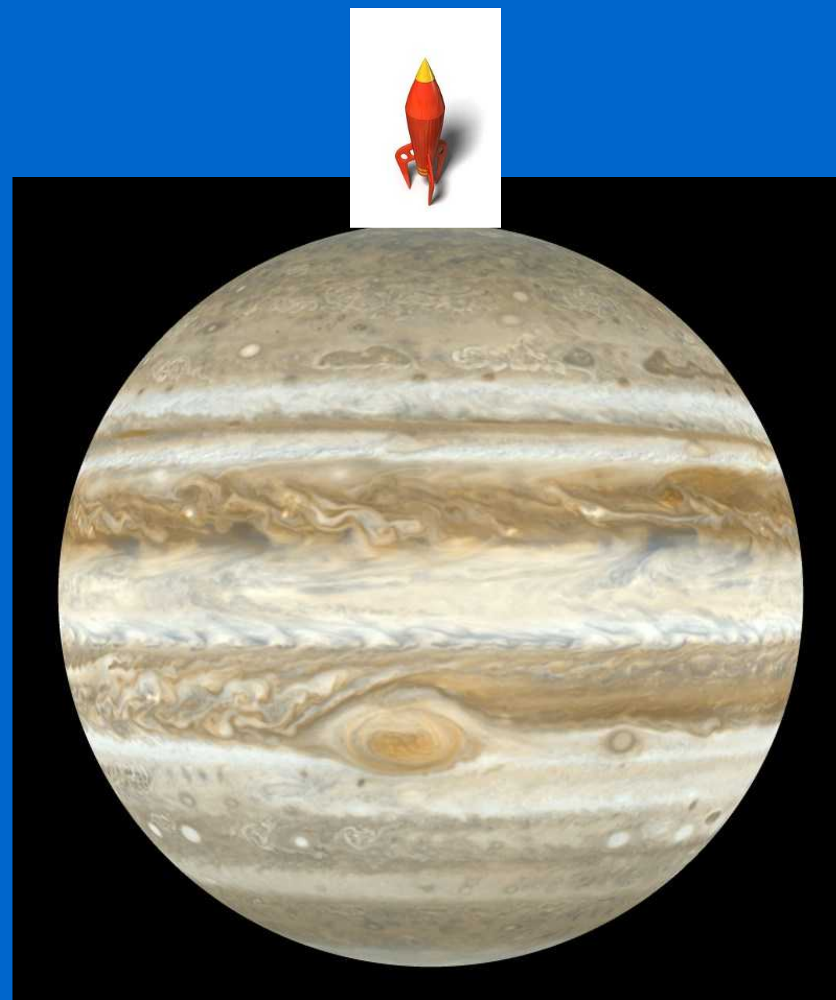
Binding energies = “how tightly a system is bound together”

or alternatively, “how much energy is required to rip it apart?”

Example 1: Binding energy of a rocket on the surface of a planet



Moon:
escape velocity
2.4 km/s



Jupiter:
escape velocity 59.5 km/s

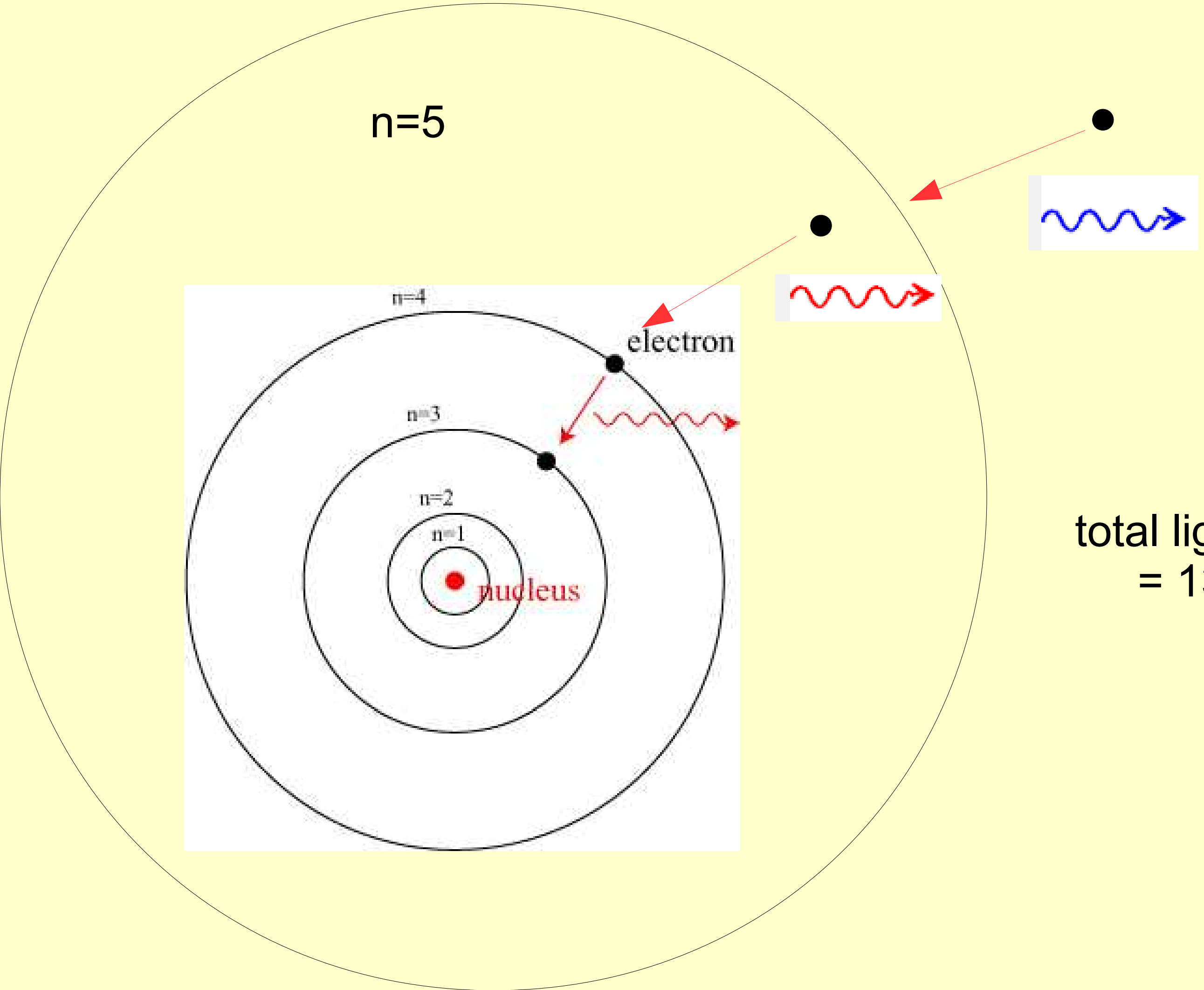
Sun:
escape velocity
617 km/s

$V_e = \sqrt{(2GM/r)}$ i.e. the larger the planet mass M, the more tightly the rocket is bound

Example 2: Binding energy of hydrogen atom

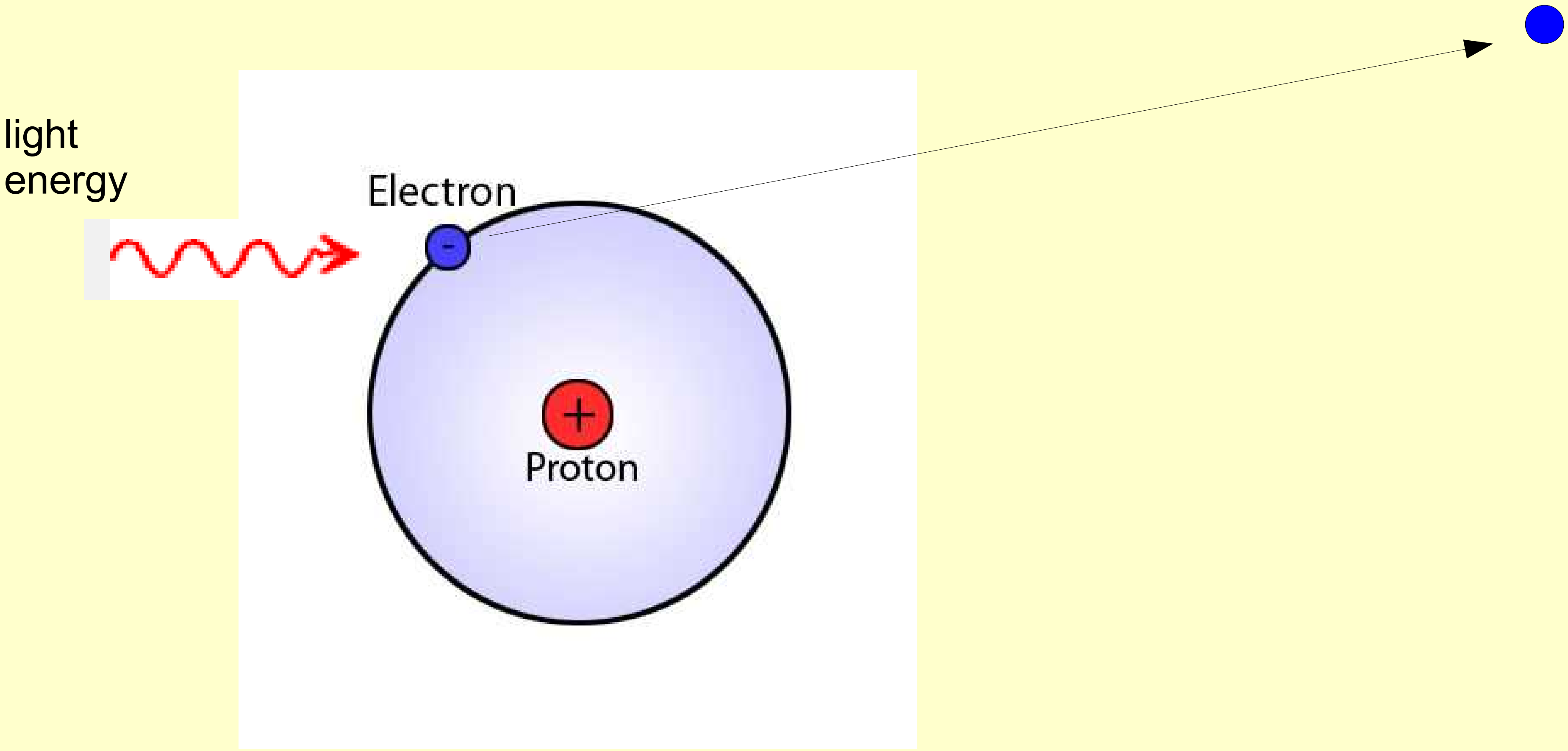
Let's assemble a hydrogen atom from a proton and an electron, initially far, far apart

The negative electron is attracted to the positive proton by the attractive electrical force.
As it spirals in to closer and closer orbits, it loses energy and radiates that energy away in the form of light.



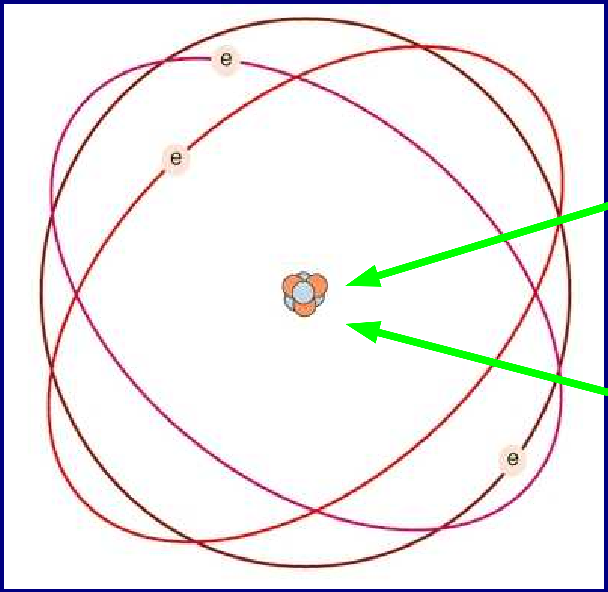
total light energy released
= 13.6 electron-volts (13.6 eV)

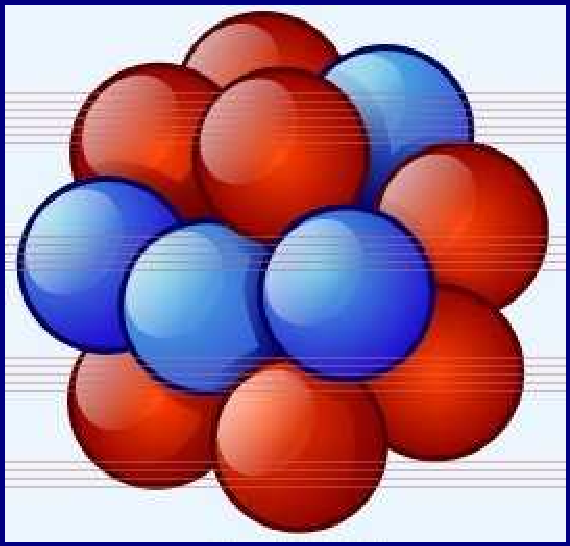
We can also do the reverse reaction, by tearing the electron away from the proton in the hydrogen atom. How much energy do we need to supply to tear the hydrogen atom apart? 13.6 eV



So we say that the binding energy of the hydrogen atom is 13.6 eV.

Now consider nuclear binding energies -- how tightly are the protons and neutrons in a nucleus bound together? How much energy does it take to tear them apart?





The nucleus is about 45,000 times smaller in diameter than an atom...like a pea in a football stadium.

Even though the atomic nucleus is so tiny compared to the entire atom, it contains 99.97% of the mass of the atom.

The nuclear matter is extremely dense – a teaspoon full would have a mass of 460 million metric tons!

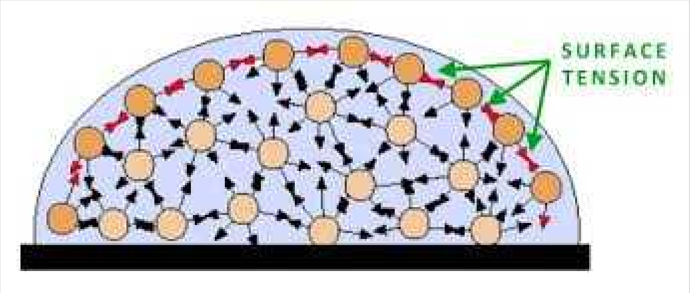
There are two types of forces that are important for determining how tightly a nucleus is bound together.

1. The repulsive force between the positively charged protons, which tends to make the nucleus fly apart.

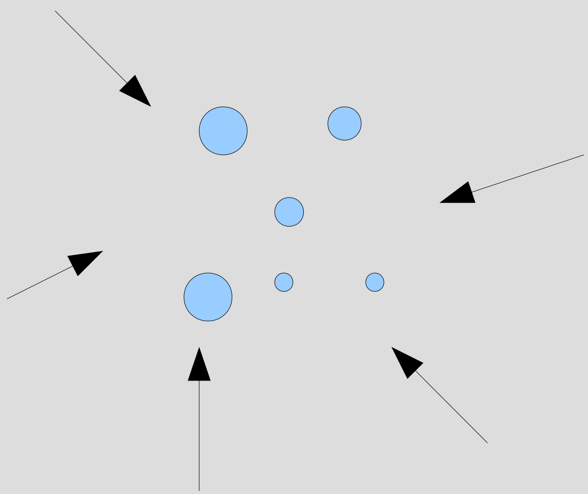
2. The short-range attractive force between all the protons and neutrons, which holds the nucleus together.

You can think of the nucleus like a drop of liquid, like water.
Water molecules naturally attract each other.

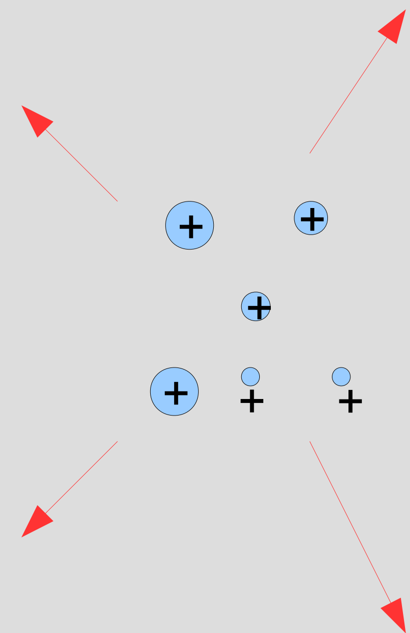
Small water droplets want to coalesce into larger drops,
to allow as many water molecules as possible to “link together” with its neighbours. Since the molecules at the surface don't have any neighbours on one side, coalescing into bigger drops reduces the percentage of molecules at the surface.



Now suppose our drops of liquid are not electrically neutral, but have a positive charge, just like an atomic nucleus. A drop of liquid bearing positive charge can't afford to get too big, because as you cram more and more positive charge close together, the positive charges repel each other more and more strongly.



Molecular attraction wants to coalesce the drops together.

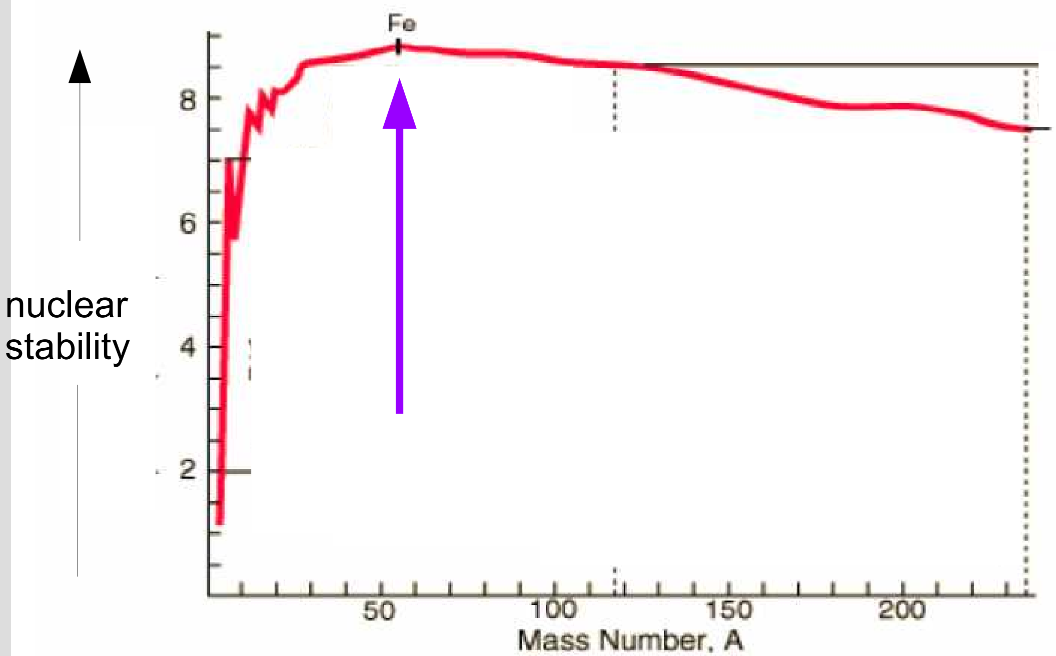


But electric repulsion wants to push them apart.

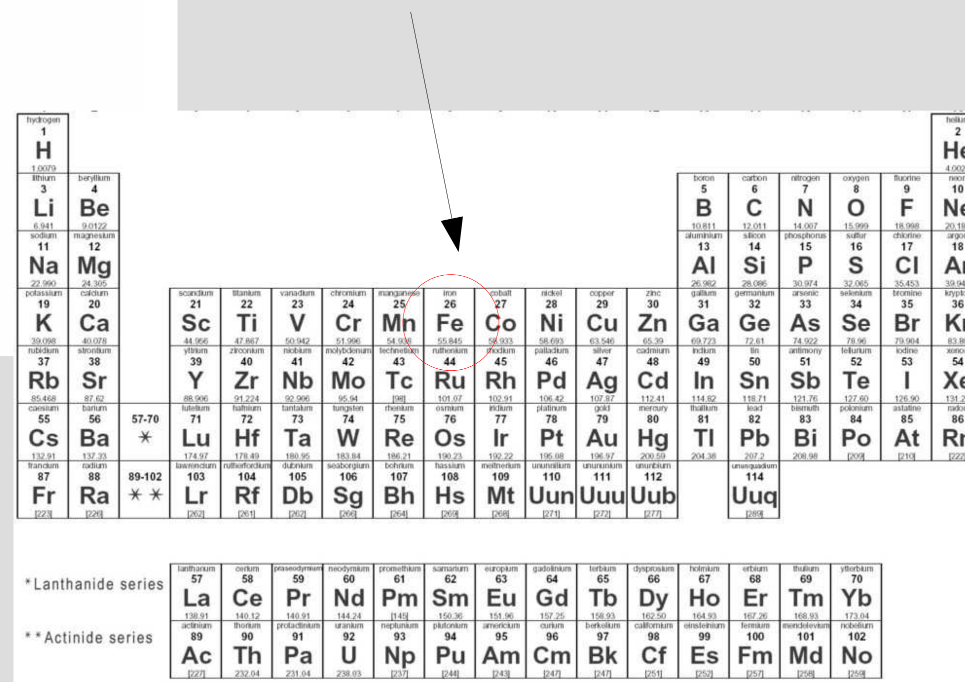
There must be some optimum size between very small and very large drops where the liquid drop is the <u>most</u> stable.

The atomic nucleus is exactly the same. It behaves like an positively-charged liquid drop. Very small nuclei want to fuse together to
be bigger to achieve greater nuclear stability, but becoming too big means stronger repulsion and less stability.
The most stable nucleus occurs at iron (not too big, not too small).

Iron is the most stable nucleus



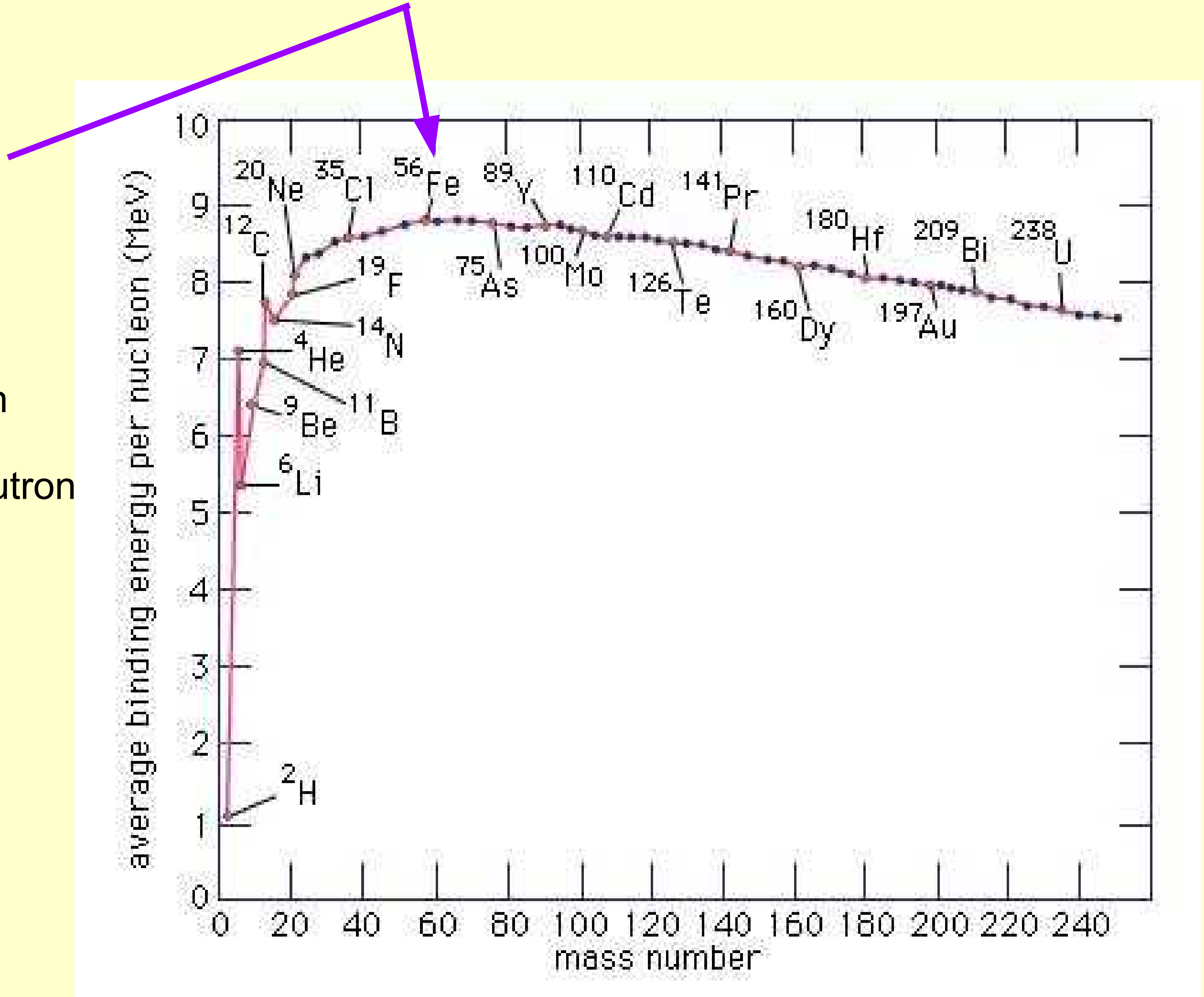
Iron element 26

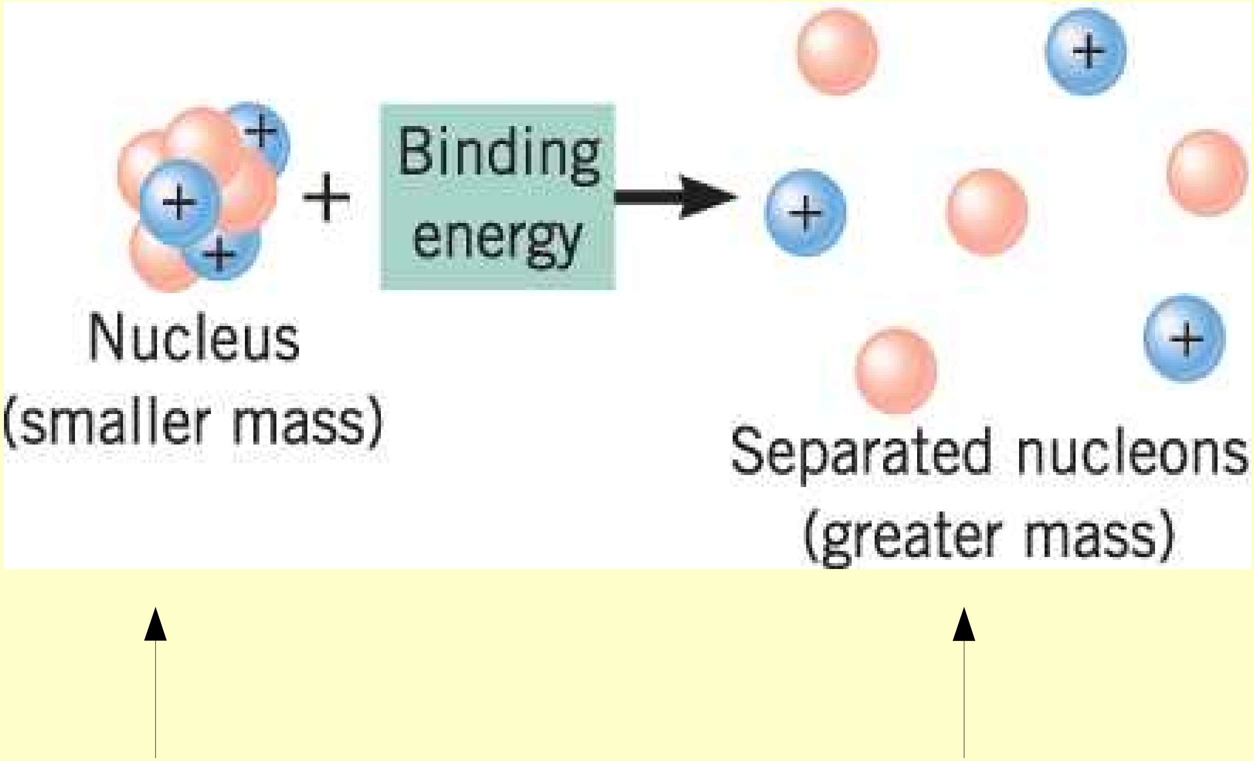


Average binding energy of 8.5 million eV (MeV) for each proton and neutron in an iron nucleus

-- that's how much energy it takes to rip a proton or neutron out of a nucleus

about 600,000 x larger than the 13.6 eV to rip an electron out of a hydrogen atom



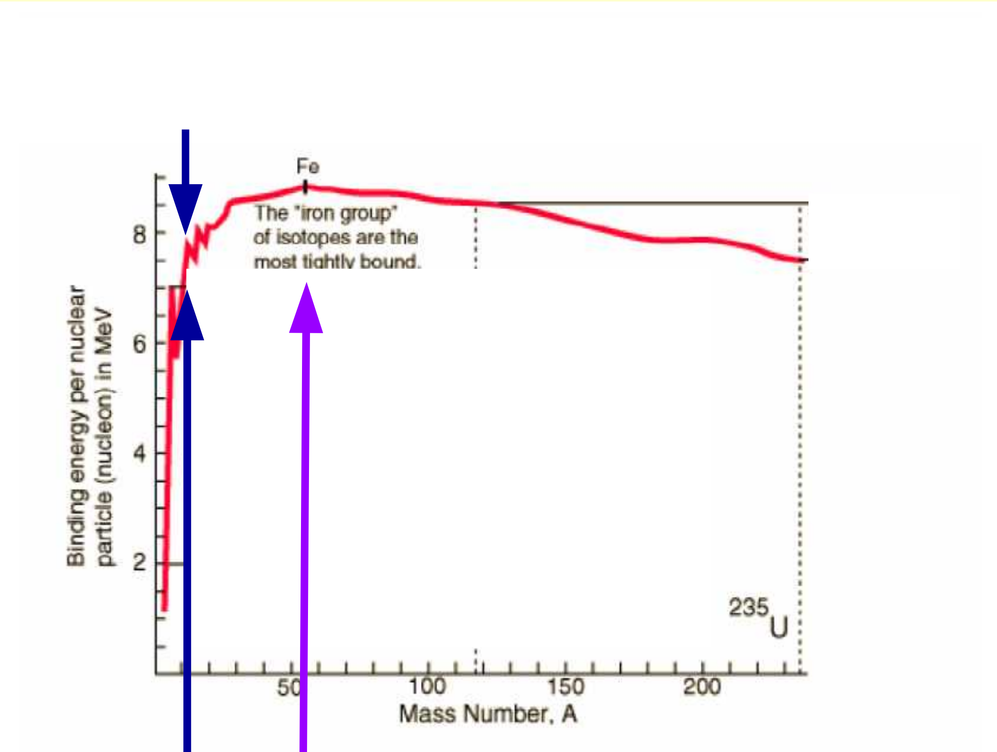


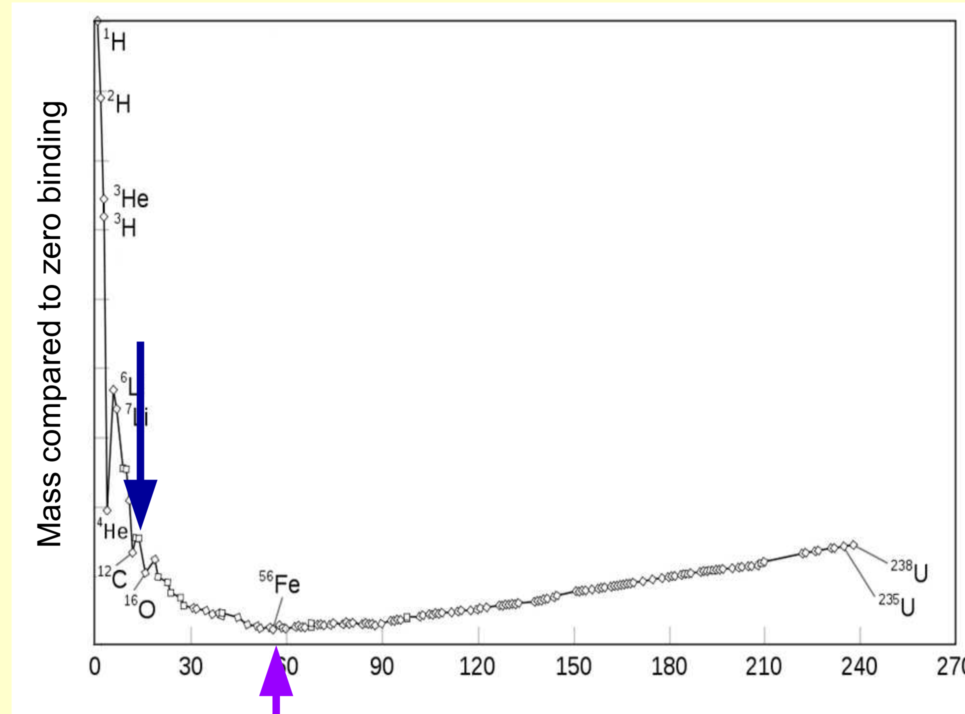
This system has less energy and thus less mass, according to $E=mc^2$

This system has more energy and thus more mass.

An atomic nucleus has <u>less mass</u> than the sum of the masses of the individual protons and neutrons that make up the nucleus, because of the nuclear binding energy.

Are nuclear binding energies large enough that we can measure a decrease in mass?





Mass number A
(number of protons+neutrons

Fe nucleus: largest average binding energy (most tightly bound)
so most energy released when nucleus is assembled,

so lowest average energy content

so from $m = E^2 / c$ this has the lowest mass compared to what you would expect if there were zero nuclear binding.

Carbon nucleus: not quite as large an average binding energy, mass decrease not quite as large as Fe

Consult your favorite chemistry book and look up the masses of atoms

^{12}C atom has 6 protons 6 electrons 6 neutrons Mass = 12.000 amu

hydrogen atom has 1 proton 1 electron

^{12}C atom has the same particles as 6 hydrogen atoms + 6 neutrons

hydrogen atom has 1 proton 1 electron Mass=1.00728 amu

neutron Mass=1.00866 amu

6 hydrogen atoms + 6 neutrons Mass = 6 x 1.00728 + 6 x 1.00866 = 12,09564 amu

These are not the same! Difference of 0.0956 amu

The ^{12}C atom weighs 0.8% **<u>less</u>** than
the sum of its constituent particles
because of the nuclear binding energy

That's easily measurable in the lab.

Our first confirmation that $E = mc^2$!

Do the same thing for Fe

^{56}Fe atom has 26 protons 26 electrons 30 neutrons Mass = 55.92068 amu

hydrogen atom has 1 proton 1 electron

^{56}Fe atom has the same particles as 26 hydrogen atoms + 30 neutrons

hydrogen atom has 1 proton 1 electron Mass=1.00728 amu

neutron Mass=1.00866 amu

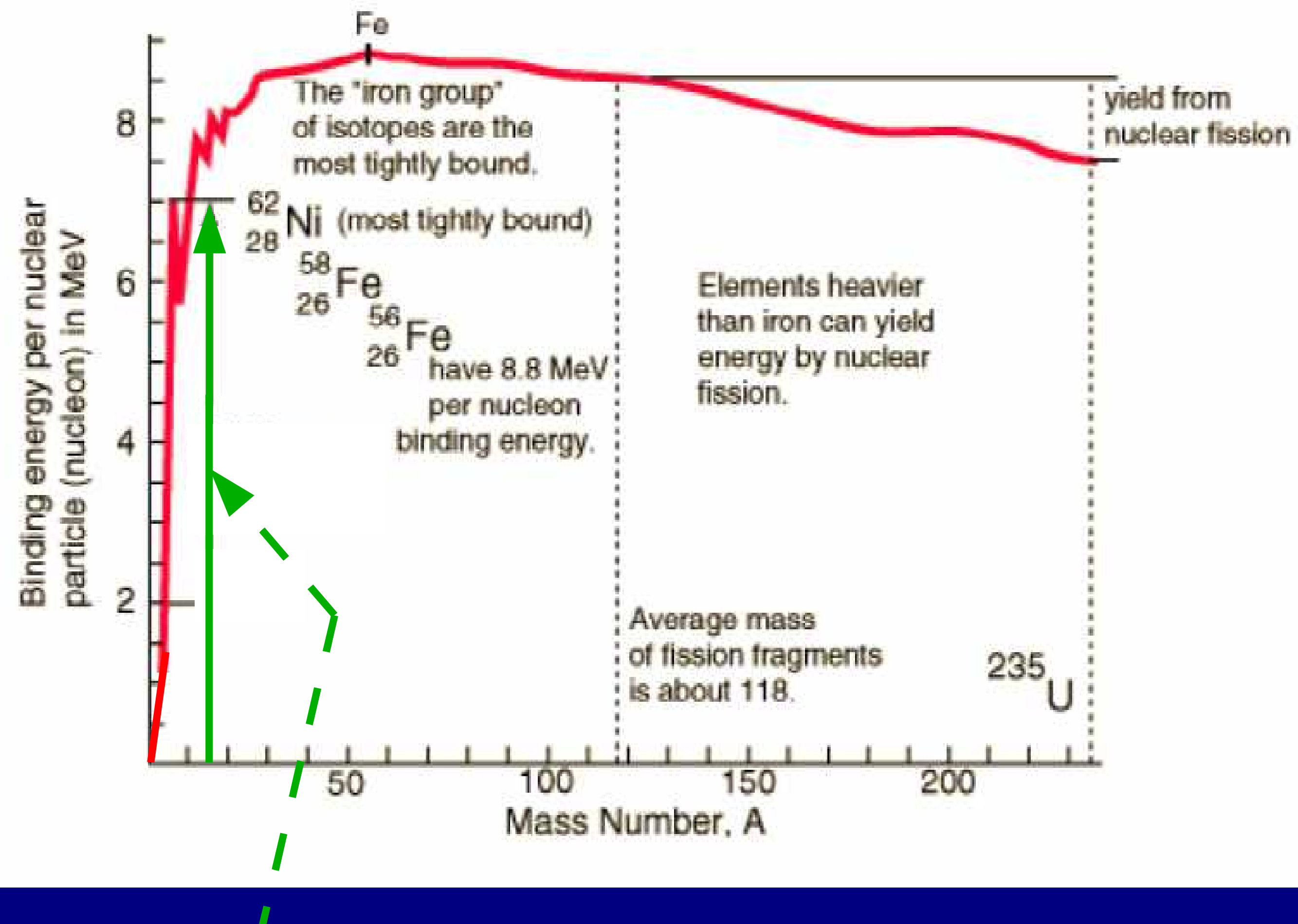
26 hydrogen atoms + 30 neutrons Mass=26 x 1.00728 + 30 x 1.00866 = 56.44908 amu

These are not the same! Difference of 0.53 MeV

The ^{56}Fe atom weighs almost 1% **<u>less</u>**
(0.94% to be exact) than
the sum of its constituent particles
because of the nuclear binding energy

This decrease is larger than for ^{12}C, as we expect,
because the Fe nucleus is more tightly bound
than the C nucleus.

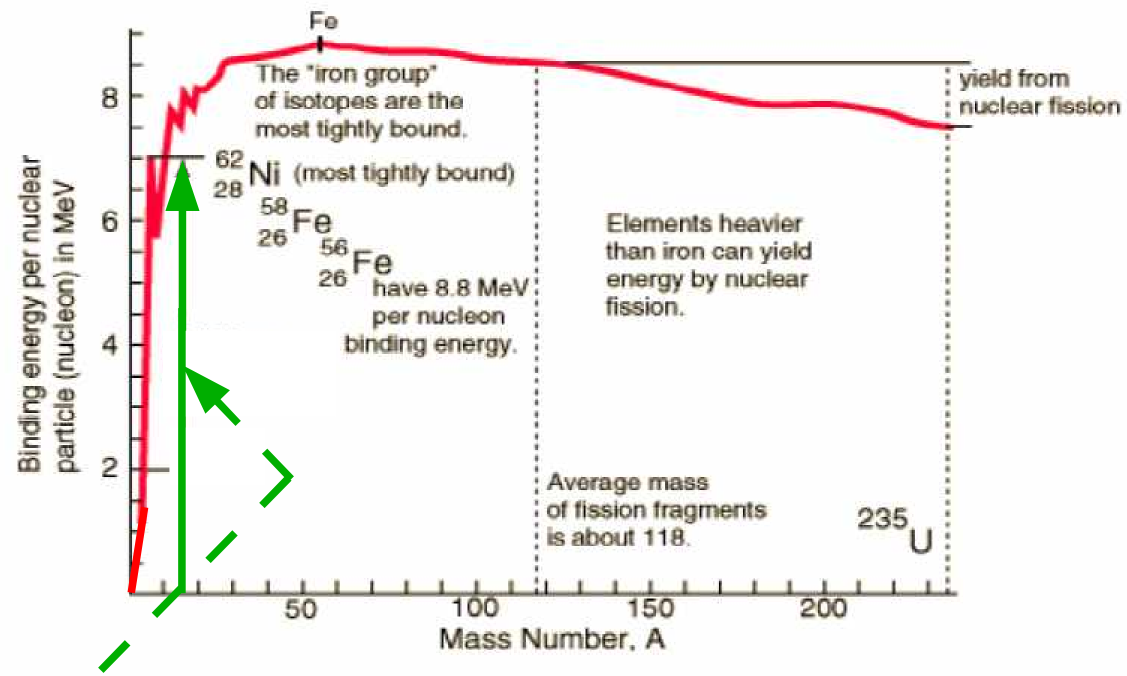
Fission and fusion can yield energy



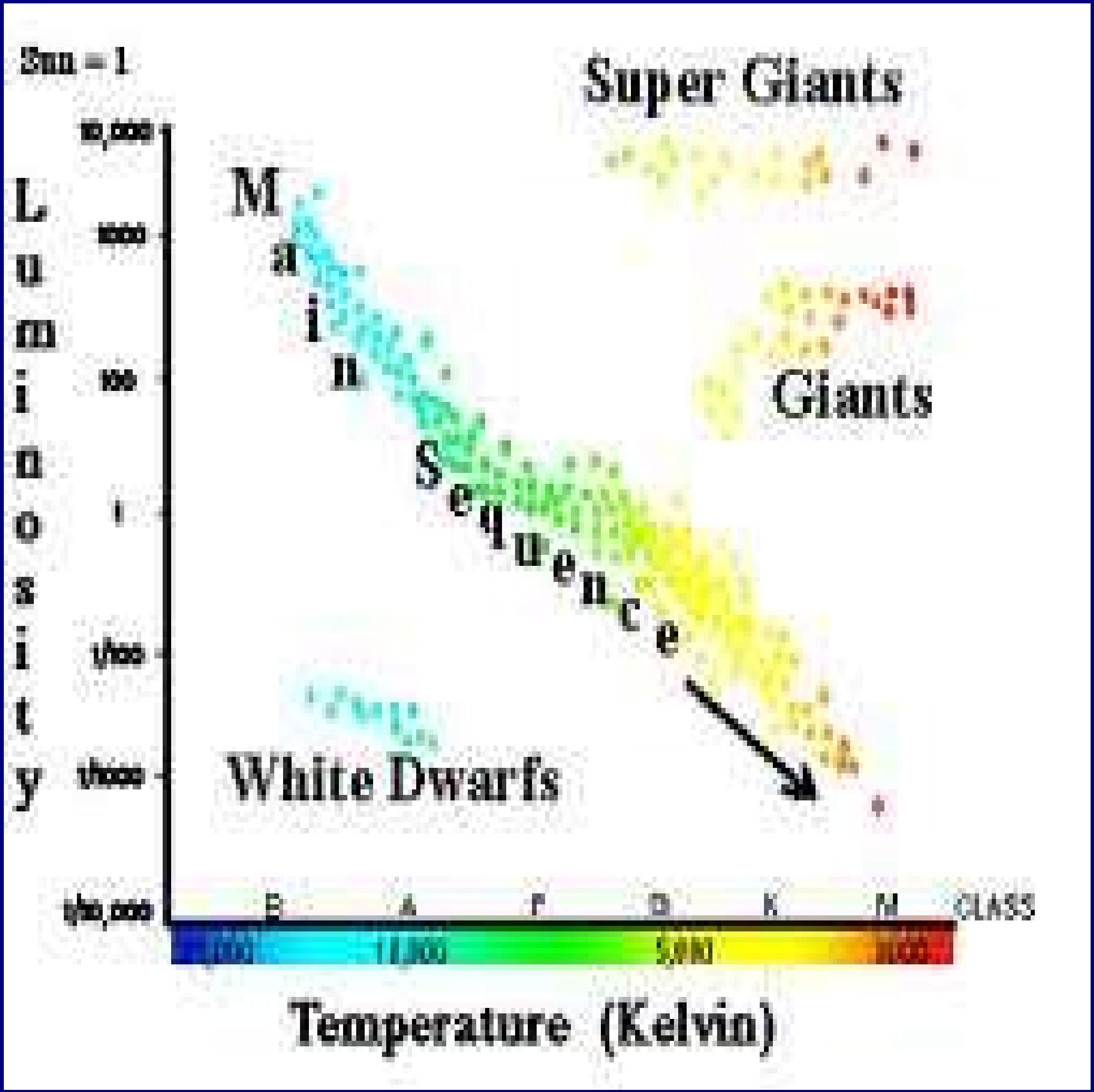
going from A=1 to A=4, the average binding energy per nucleon increases from 0 to 7 MeV

Fusion reaction 4 p $\rightarrow$ $^4He + 2\ e^+ + 2\ \nu_e$ liberates ~ 4 x 7 = 28 MeV
THIS REACTION PRODUCES ENERGY IN THE SUN!

Fission and fusion can yield energy

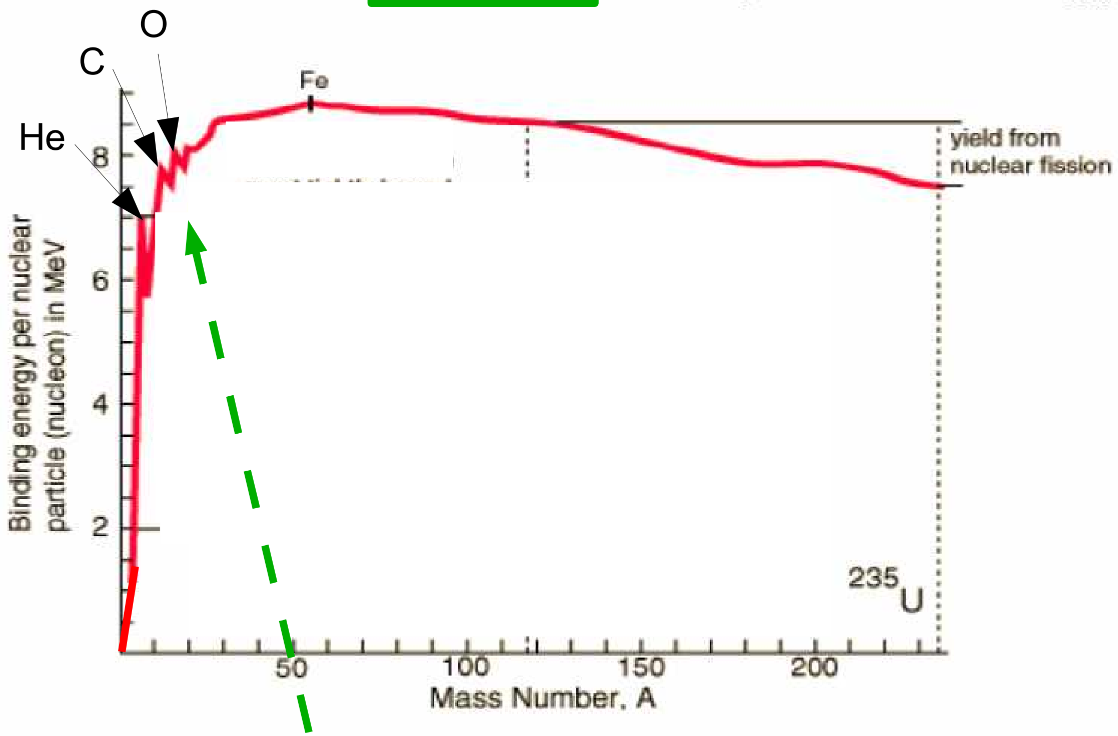


hydrogen→helium gives the biggest gain in binding energy. Stars spend most of their lives in this stage. (Main sequence stars).



“Hertzsprung-Russell diagram” for the stars in a cluster

Fission and fusion can yield energy

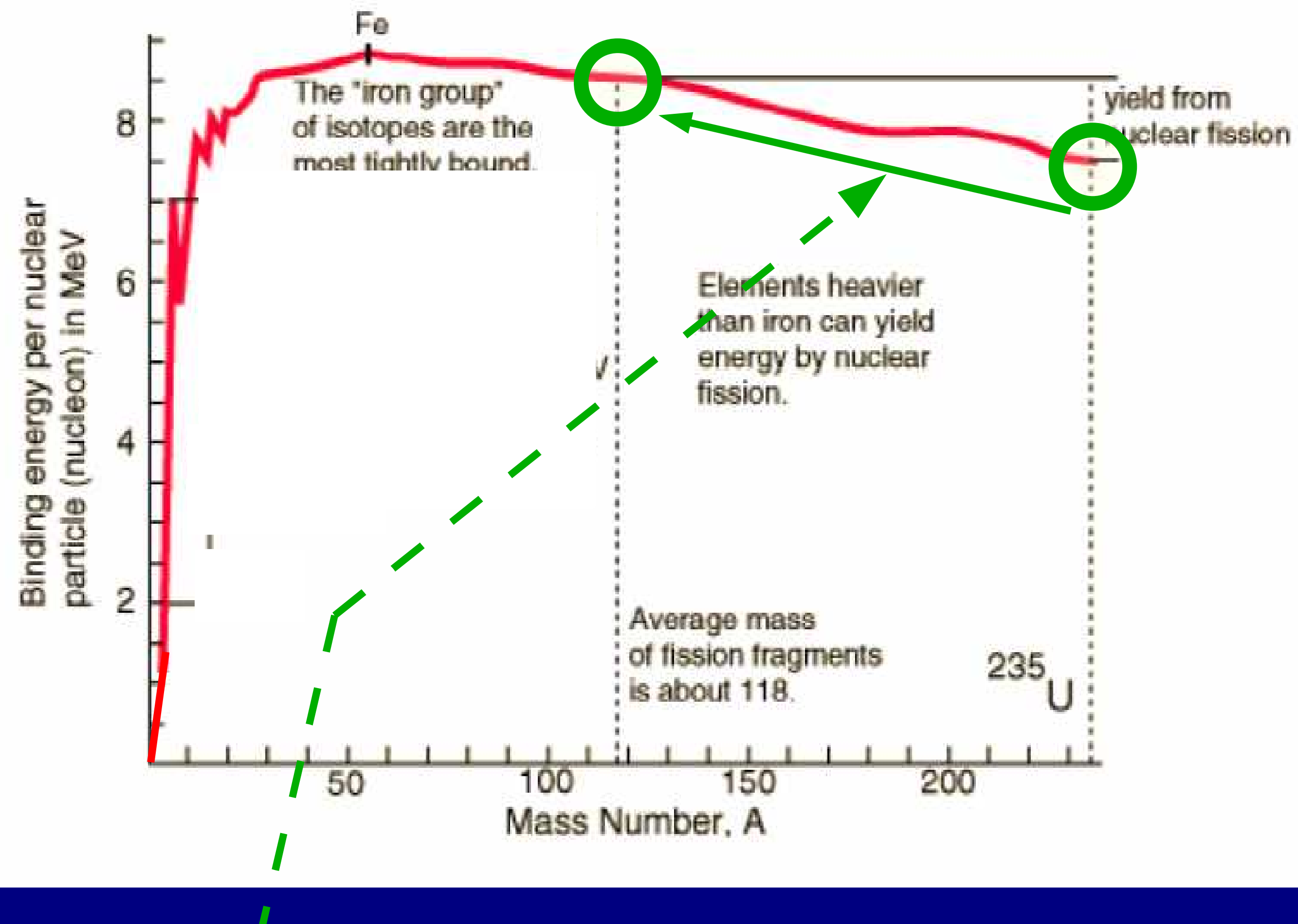


Later stages of a star's life, during which it fuses He+He+He→C, He+C→O, etc. produce far less energy and so last much shorter periods of time.

Timeline for a 25 solar mass star

Hydrogen burning	7 Myr
Helium burning	500 kyr
Carbon burning	600 yr
Neon burning	1 yr
Silicon burning	1 day
Core collapse	<1 second

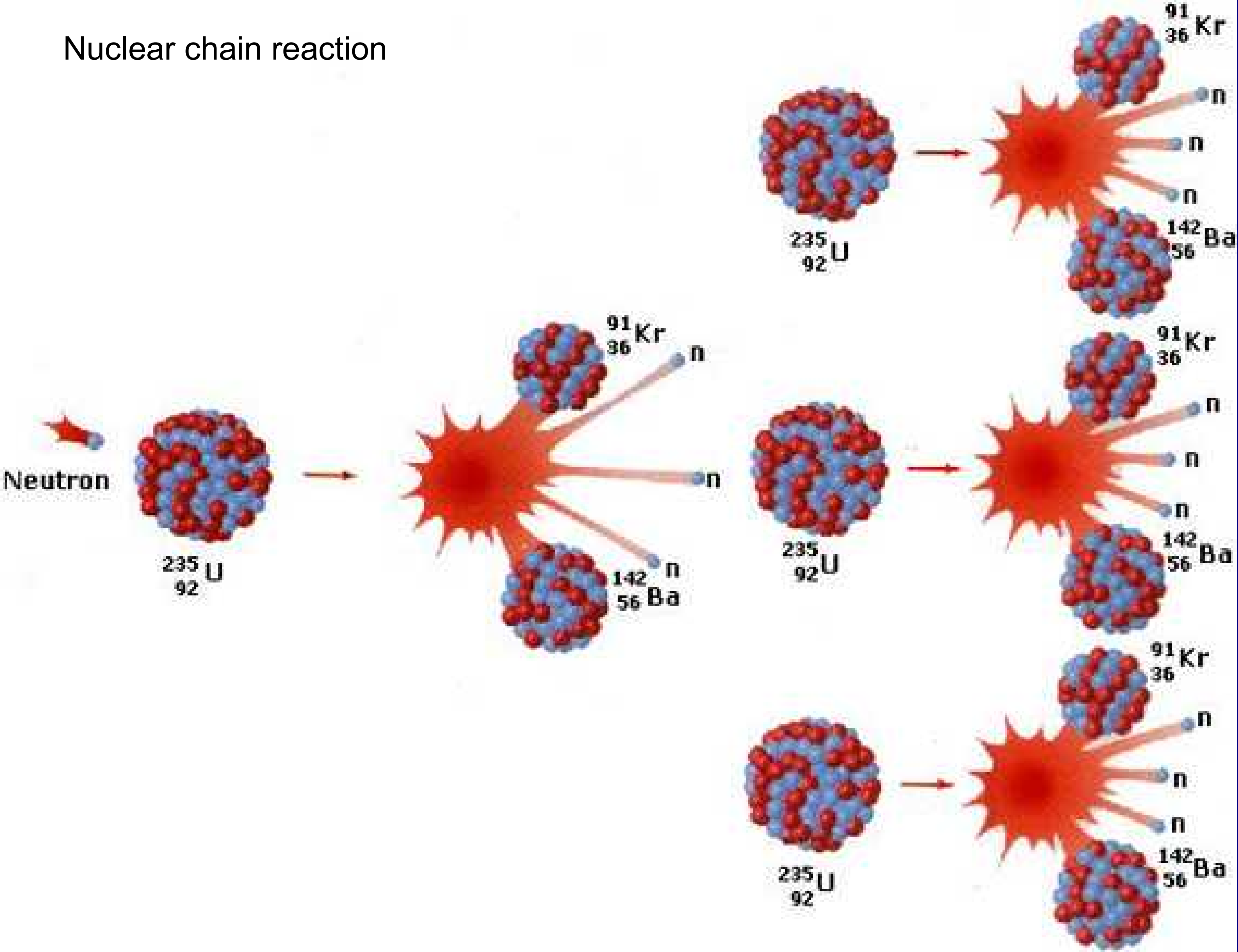
Fission and fusion can yield energy



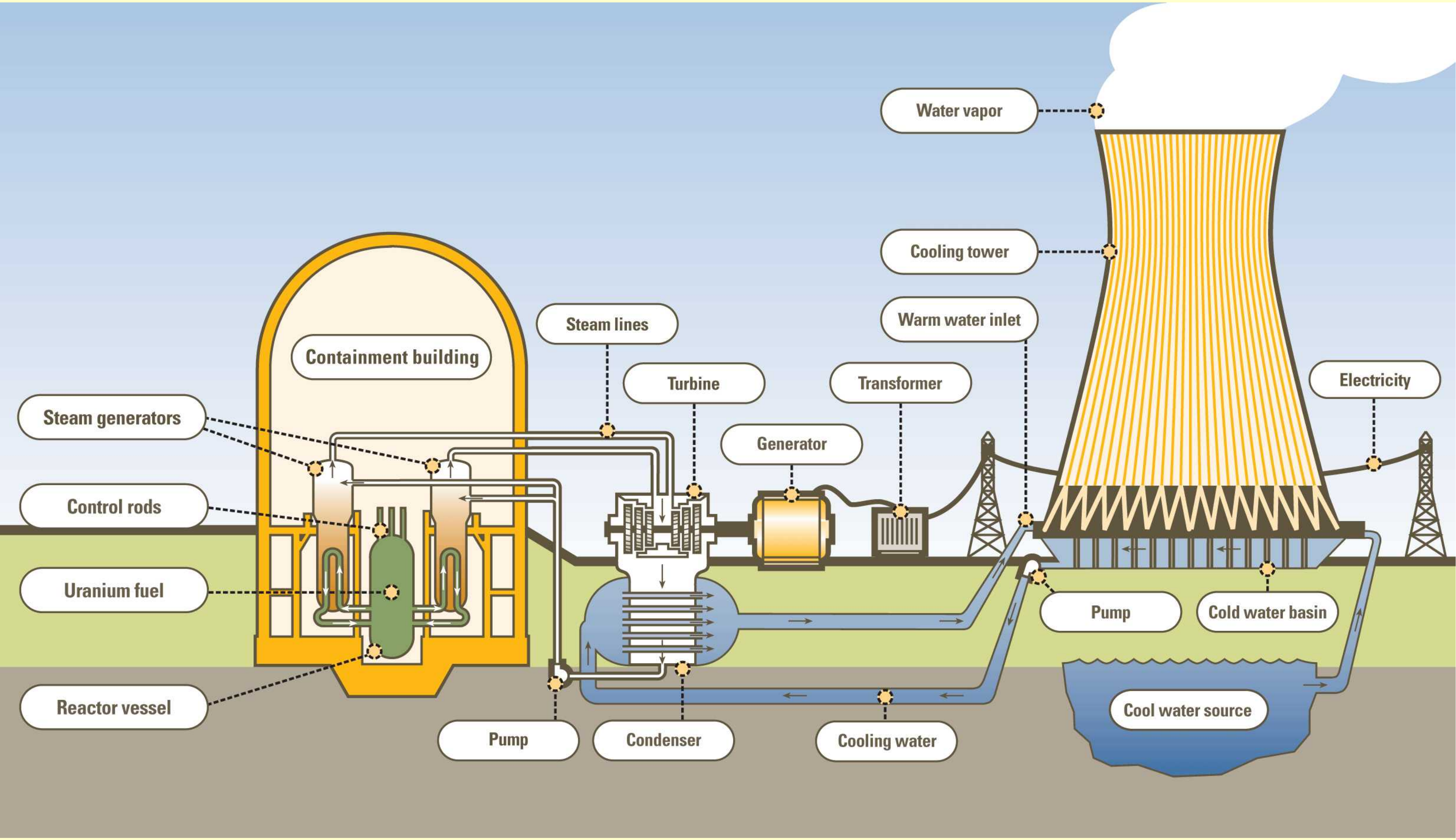
going from A=235 to A=118, the average binding energy per nucleon increases by ~ 1 MeV

Fission of ^{235}U into 2 equal fragments gives about 235 x 1 = 235 MeV.
THIS REACTION PRODUCES ENERGY IN NUCLEAR REACTORS.

Nuclear chain reaction



If the chain reaction has a steady, controlled number of neutrons, we have a nuclear fission reactor, which can produce electricity



If the number of neutrons, and hence the number of fissions, increases exponentially, then we have a runaway chain reaction, which results in an explosion. When it was realized that this could be the basis for an atomic bomb, and that the Nazis may be working on such a bomb, Einstein signed a letter to US president Roosevelt in August 1939.

Albert Einstein
Old Grove Rd.
Nassau Point
Peconic, Long Island

August 2nd, 1939

F.D. Roosevelt,
President of the United States,
White House
Washington, D.C.

Sir:

Some recent work by E.Fermi and L. Szilard, which has been communicated to me in manuscript, leads me to expect that the element uranium may be turned into a new and important source of energy in the immediate future. Certain aspects of the situation which has arisen seem to call for watchfulness and, if necessary, quick action on the part of the Administration. I believe therefore that it is my duty to bring to your attention the following facts and recommendations:

In the course of the last four months it has been made probable - through the work of Joliot in France as well as Fermi and Szilard in America - that it may become possible to set up a nuclear chain reaction in a large mass of uranium,by which vast amounts of power and large quantities of new radium-like elements would be generated. Now it appears almost certain that this could be achieved in the immediate future.

This new phenomenon would also lead to the construction of bombs, and it is conceivable - though much less certain - that extremely powerful bombs of a new type may thus be constructed. A single bomb of this type, carried by boat and exploded in a port, might very well destroy the whole port together with some of the surrounding territory. However, such bombs might very well prove to be too heavy for transportation by air.

-2-

The United States has only very poor ores of uranium in moderate quantities. There is some good ore in Canada and the former Czechoslovakia, while the most important source of uranium is Belgian Congo.

In view of this situation you may think it desirable to have some permanent contact maintained between the Administration and the group of physicists working on chain reactions in America. One possible way of achieving this might be for you to entrust with this task a person who has your confidence and who could perhaps serve in an inofficial capacity. His task might comprise the following:

a) to approach Government Departments, keep them informed of the further development, and put forward recommendations for Government action, giving particular attention to the problem of securing a supply of uranium ore for the United States;

b) to speed up the experimental work,which is at present being carried on within the limits of the budgets of University laboratories, by providing funds, if such funds be required, through his contacts with private persons who are willing to make contributions for this cause, and perhaps also by obtaining the co-operation of industrial laboratories which have the necessary equipment.

I understand that Germany has actually stopped the sale of uranium from the Czechoslovakian mines which she has taken over. That she should have taken such early action might perhaps be understood on the ground that the son of the German Under-Secretary of State, von Weizsäcker, is attached to the Kaiser-Wilhelm-Institut in Berlin where some of the American work on uranium is now being repeated.

Yours very truly,
A. Einstein
(Albert Einstein)

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and the letter then goes on to warn that Nazi Germany may be working to build such a bomb.

“The Manhattan Project” - to build an atomic bomb

“Little Boy” - a bomb using ^{235}U

64 kg of uranium

Blast equal to 15 kilotons of TNT

dropped on Hiroshima

“Fat Man” - a bomb using ^{239}Pu

6.2 kg of plutonium

Blast equal to 21 kilotons of TNT

dropped on Nagasaki

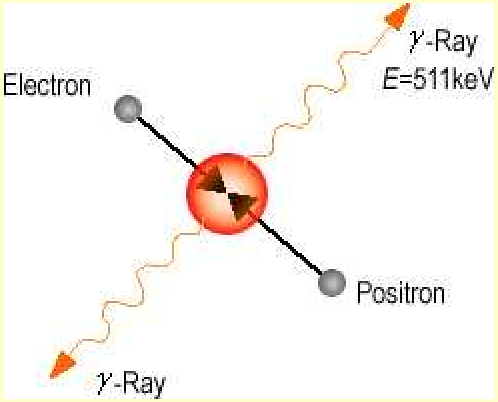
A very unfortunate application of $E=mc^2$

In nuclear fission of uranium, only about 0.1% of the mass is converted into energy. In nuclear fusion of hydrogen to helium, only 0.7% of the mass is converted to energy. (so a hydrogen bomb, which uses fusion of hydrogen, is more powerful than a uranium bomb).

Is there some process where 100% of the mass could be converted into energy? Yes – anti-matter annihilation!

Electron	Anti-electron (positron)
- charge	+ charge
m = 1 / 1823 amu	m = 1 / 1823 amu
$mc^2 = 0.511$ MeV	$mc^2 = 0.511$ MeV

When an electron and a positron meet each other, they annihilate and produce two gamma rays going in opposite directions. Mass is changed into energy!



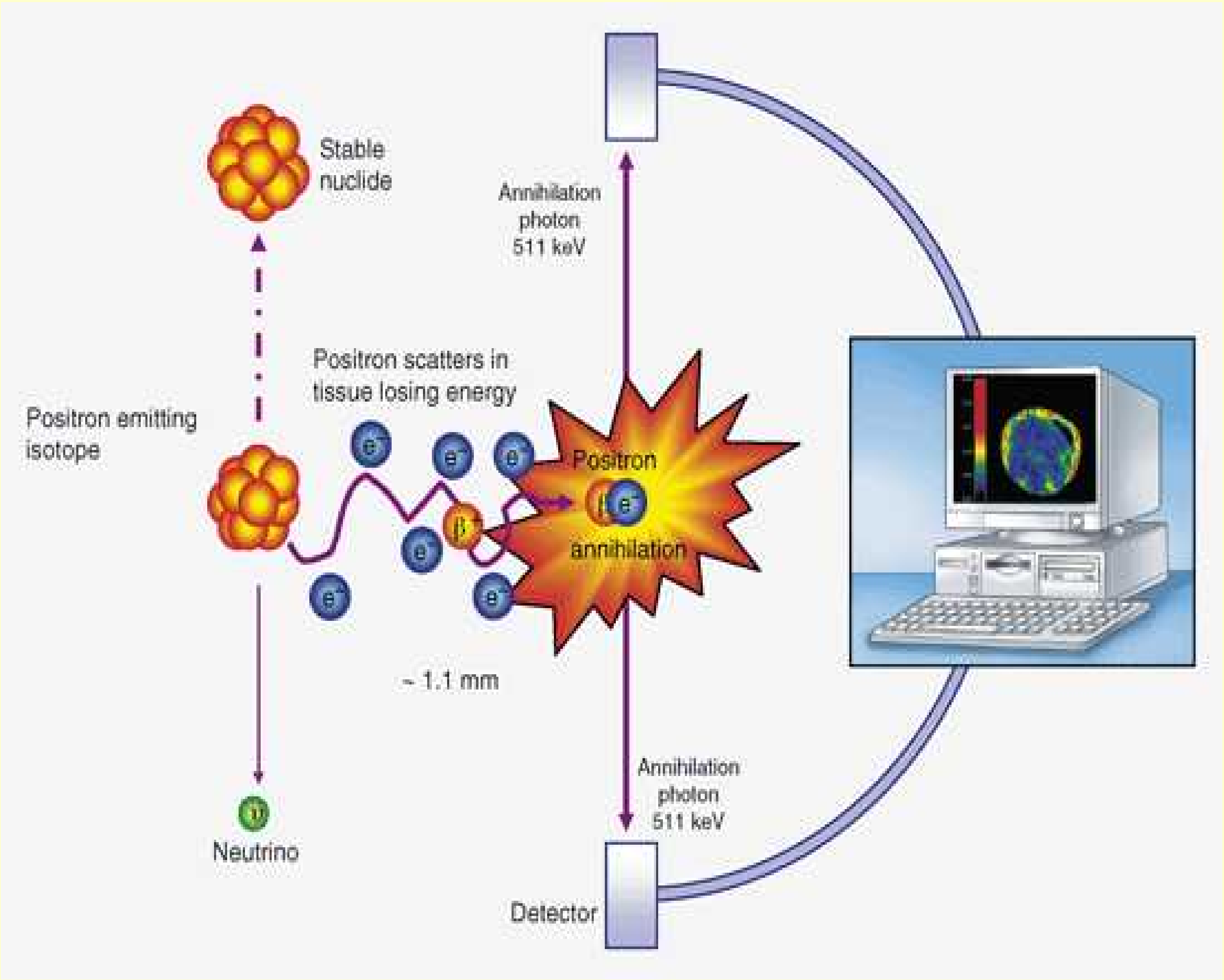
Using a cyclotron, we can produce certain radioactive isotopes that decay by emitting anti-matter electrons

e.g. ^{11}C just like normal ^{12}C in your body, but missing one neutron; half-life of 20 min.

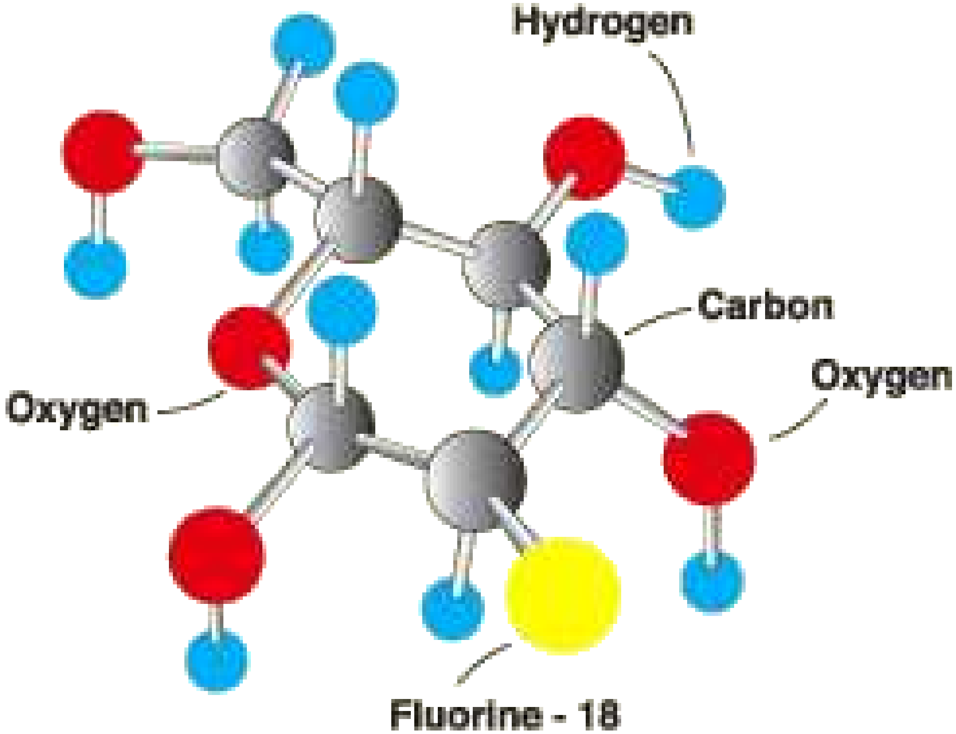
^{18}F just like the ^{19}F in your toothpaste, but missing one neutron; half-life 2 hours

These isotopes are produced at TRIUMF and used for a medical imaging technique called Positron Emission Tomography (PET Scan).

small 13 MeV cyclotron at TRIUMF for producing PET isotopes



Stable nuclide
Annihilation photon 511 keV
Positron scatters in tissue losing energy
Positron emitting isotope
Positron
annihilation
~ 1.1 mm
Annihilation photon 511 keV
Neutrino
Detector



FGD Fluoro-deoxyglucose

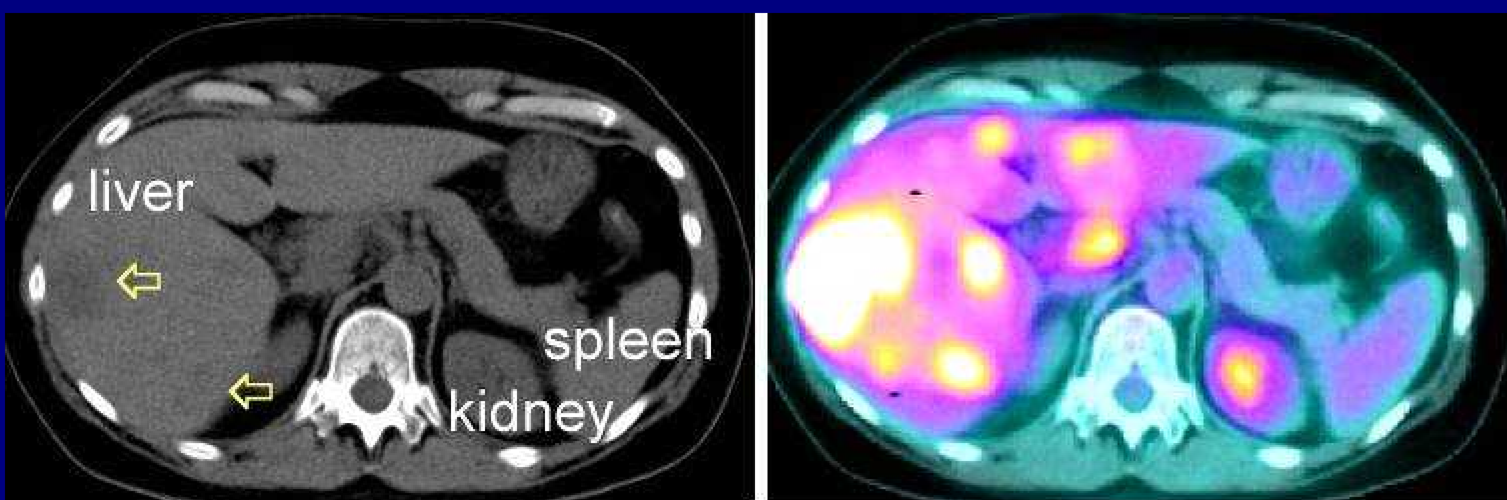
A glucose molecule with a radioactive Fluorine-18 atom attached.

Unlike a CT scan which gives information about density structures in the body

a PET scan tells about the metabolic function.

The best way to scan for metastatic cancer ... the BC Cancer agency has a cyclotron for this purpose.

A very <u>fortunate</u> application of $E = mc^2$



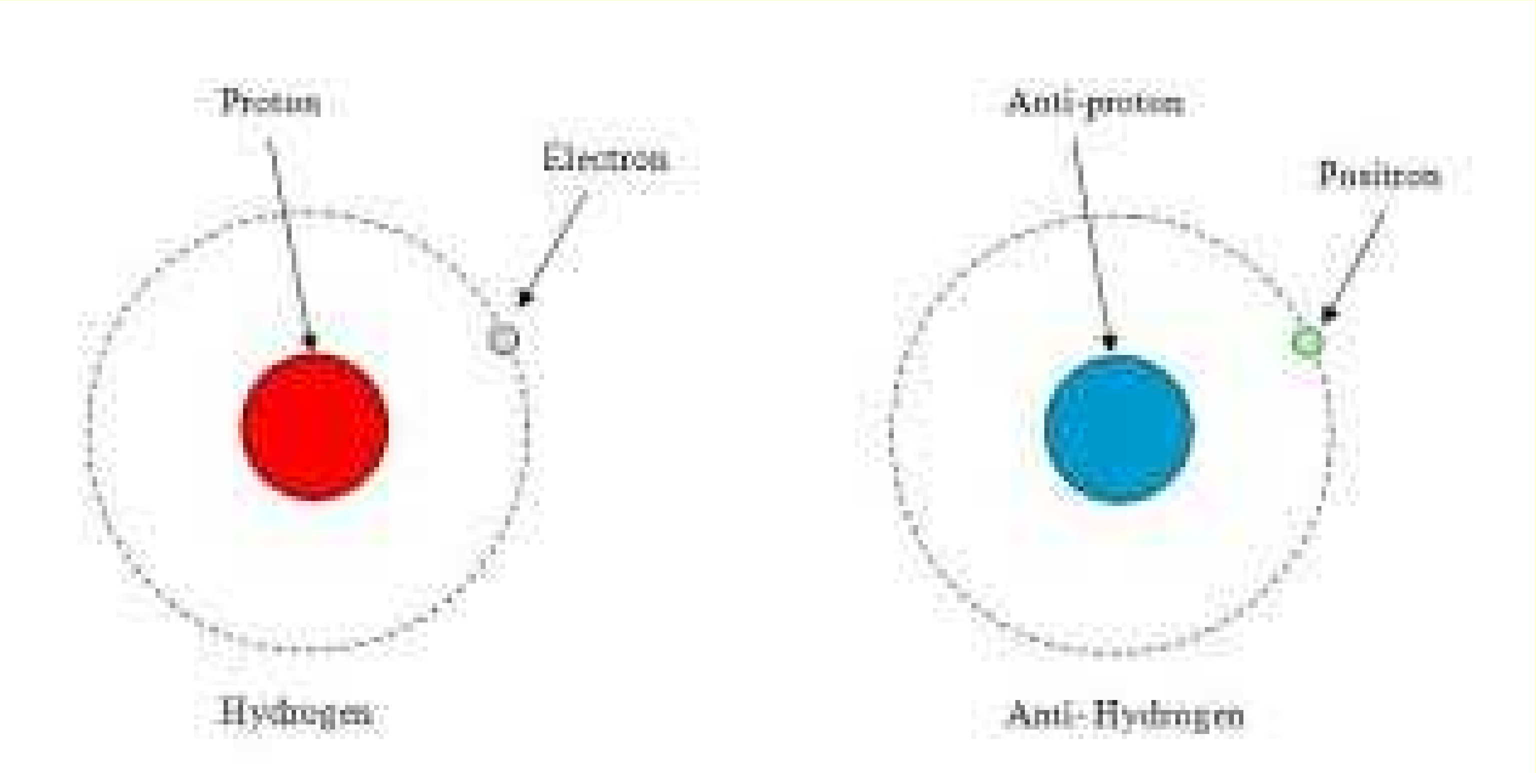
CT Scan - liver mets very subtle

PET Scan - liver mets very obvious

In principal, even more energy can be obtained if we annihilate protons with anti-protons, because they are more massive than electrons/positrons

Proton	Anti-proton
+ charge	- charge
m = 1.00728 amu	m = 1.00728 amu
mc^2 = 938 MeV	mc^2 = 938 MeV

Once you make anti-protons, you can combine them with anti-electrons to make anti-hydrogen atoms:



When these annihilate, 1877 MeV of energy will be released – far more than the 1.02 MeV released when electrons and positrons annihilate.

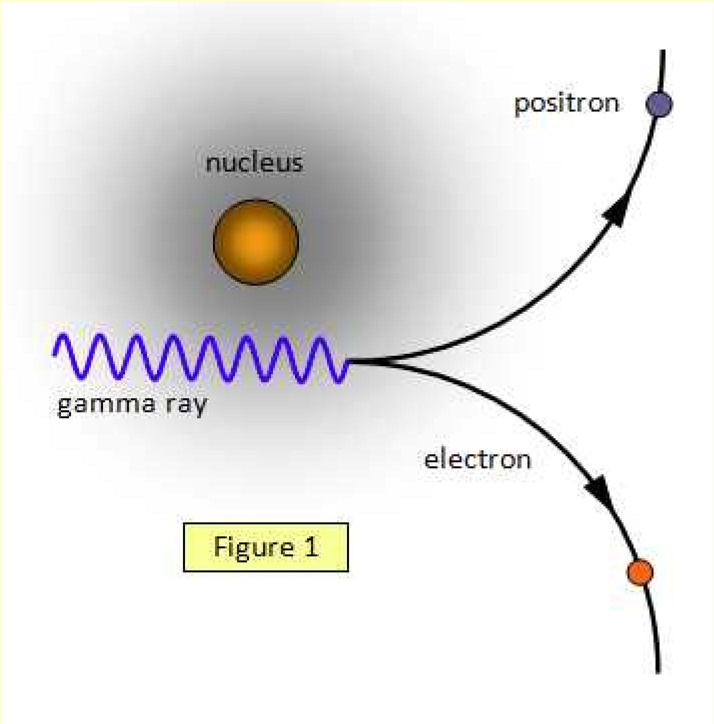
In the movie "da Vinci Code", terrorists threatens to destroy the Vatican with an anti-matter bomb from CERN (the big particle accelerator in Geneva).

Anti-protons and anti-hydrogen atoms are made at CERN, but only a few atoms at a time, and not enough to make a bomb – which is a fortunate thing!

We have looked at how we can change mass into energy
e.g. nuclear fusion, or nuclear fission, or anti-matter annihilation

But can we do the reverse and change energy into mass?
The answer is YES!

Gamma rays of energy > 1.02 MeV (like those from a radiation therapy machine at the hospital) hitting a piece of material will spontaneously turn into an electron and a positron (a positive electron, which is an anti-matter electron).



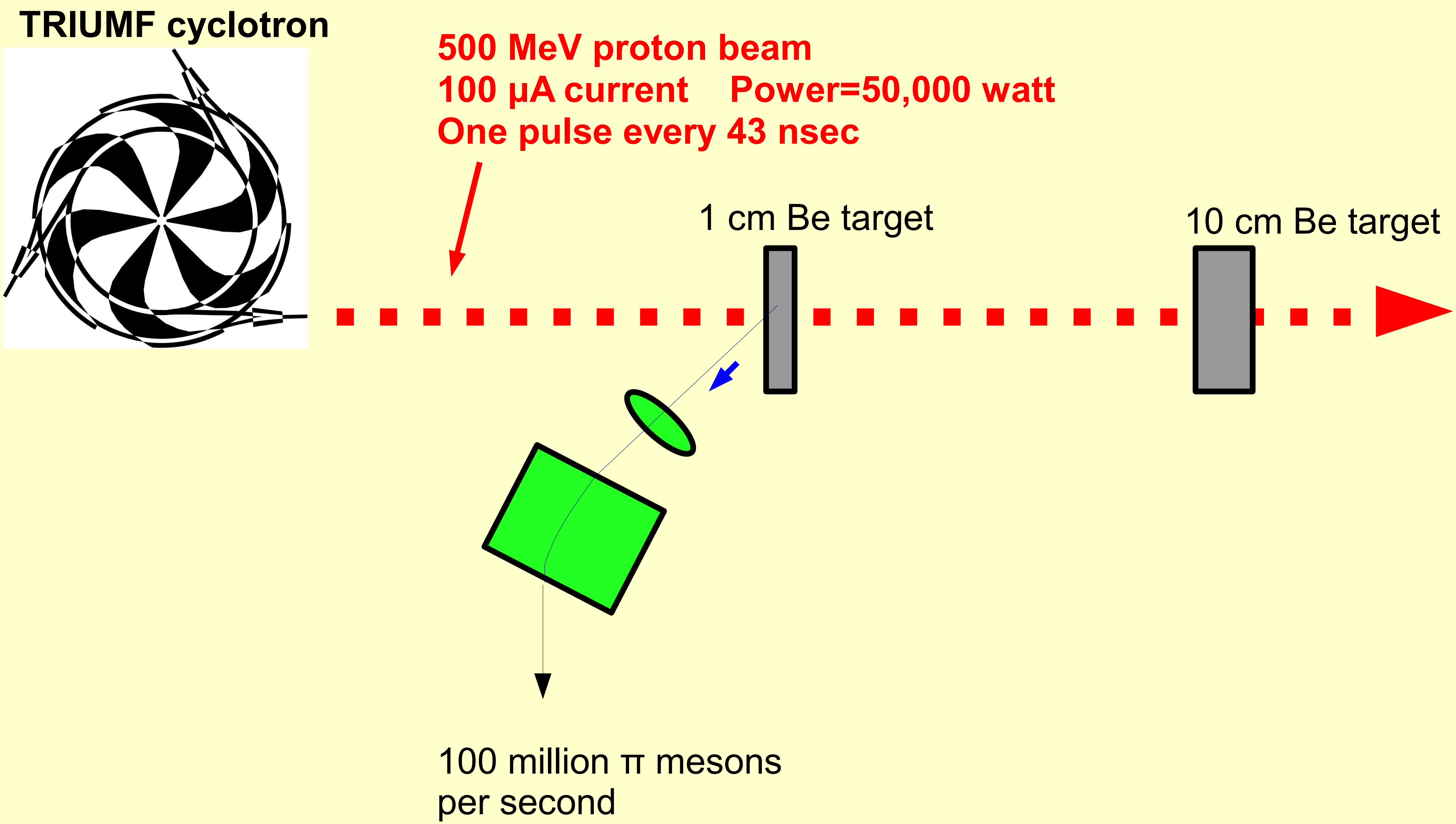
the gamma ray (energy) disappears

two new particles appear

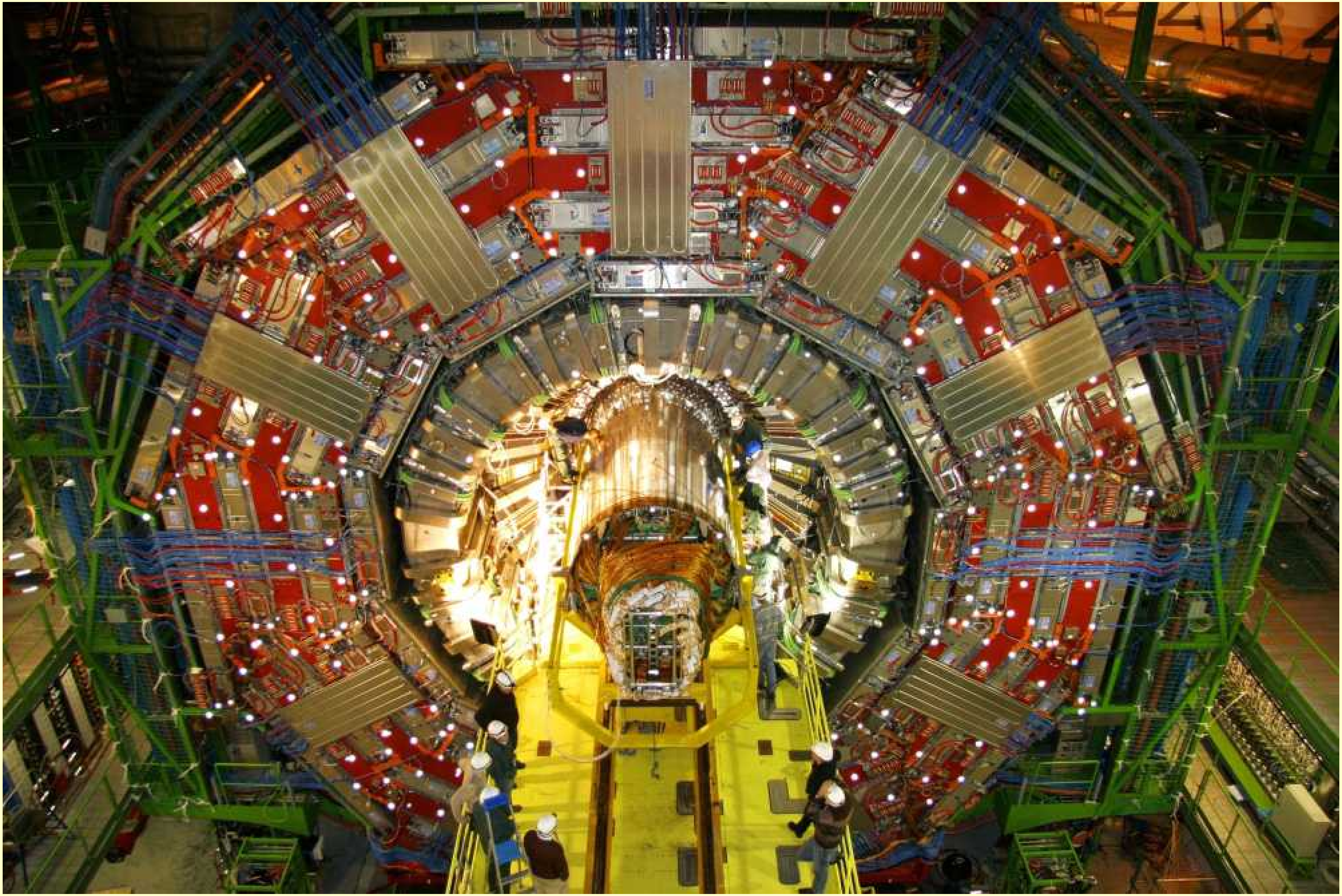
energy changed into matter according to $E = mc^2$

The conversion of energy to matter is used at particle accelerators to produce new particles that didn't exist before

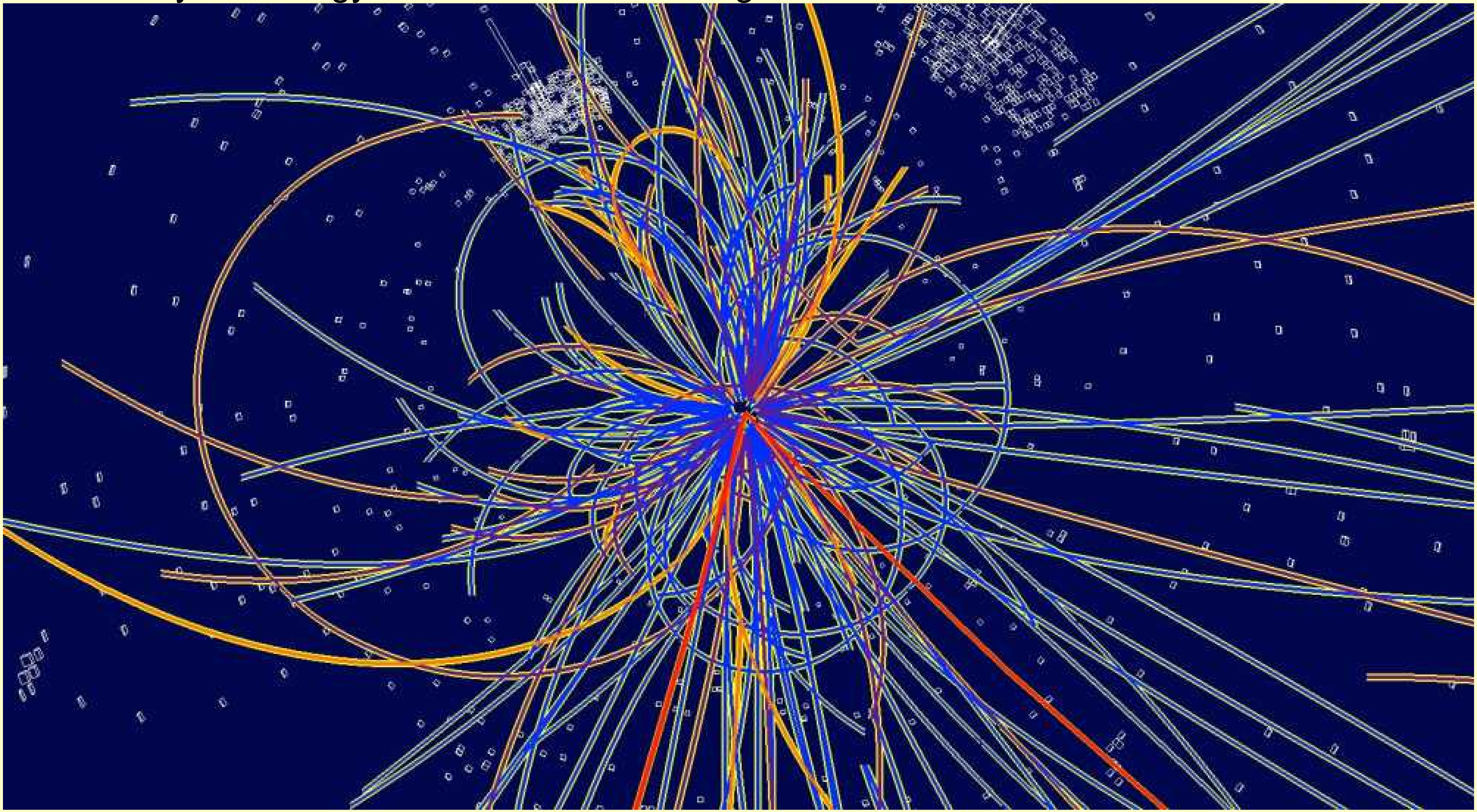
e.g. here at TRIUMF, we use the cyclotron to produce subatomic particles called π mesons



The Large Hadron Collider – the world's highest energy particle accelerator
Two proton beams, each of energy 7 Trillion eV, collide head on



tracks in the detector left by the dozens of particles made when two 7 TeV protons collide head-on. None of these particles existed before the collision – they were created by the energy of 7 TeV + 7 TeV turning into mass !



In 2012, the LHC observed the Higgs boson, with a mc^2 = 125 billion electron volts, which is about 133 times heavier than a proton and about as heavy as a tin nucleus. Previous accelerators didn't have enough energy to produce these.

SUMMARY:

$E = mc^2$ tells us that we can convert mass into energy, and vice versa.

Because c^2 is a large number, a small amount of mass becomes a huge amount of energy.

Conversely, it takes a huge amount of energy to create a small amount of mass.

Since energy and mass can be converted to each other, it is no longer adequate to talk about "conservation of mass" or "conservation of energy", since mass and energy are not individually conserved any more.

Rather, it is the sum of mass + energy that is conserved, so we should instead talk about "conservation of mass-energy".